

MATHEMATICS

Test Admission Ticket No.

Question Booklet Version Code
A
<i>(Write this Code on your OMR Answer sheet)</i>

Question Booklet Sr. No.
84681
<i>(Write this Number on your OMR Answer sheet)</i>

OMR Serial Number

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- * Before commencing the examination, please verify that all pages are printed correctly. If not, please draw the attention of your room invigilator for further assistance.
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1. If $\vec{u} = \vec{a} - \vec{b}$, $\vec{v} = \vec{a} + \vec{b}$, and $|\vec{a}| = |\vec{b}| = 2$, then $|\vec{u} \times \vec{v}|$ is

a) $2\sqrt{16 - (\vec{a} \cdot \vec{b})^2}$

b) $2\sqrt{4 - (\vec{a} \cdot \vec{b})^2}$

c) $\sqrt{16 - (\vec{a} \cdot \vec{b})^2}$

d) $\sqrt{4 - (\vec{a} \cdot \vec{b})^2}$

$$\begin{aligned} |\vec{u} \times \vec{v}| &= |(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})| = 2\vec{a} \times \vec{b} = 2|\vec{a}||\vec{b}|\sin\theta \\ &= 2|\vec{a}||\vec{b}|\sqrt{1 - \cos^2\theta} \\ &= 2\sqrt{|\vec{a}|^2|\vec{b}|^2 - |\vec{a}||\vec{b}|\cos^2\theta} \\ &= 2\sqrt{4 \times 4 - (\vec{a} \cdot \vec{b})^2} \\ &= 2\sqrt{16 - (\vec{a} \cdot \vec{b})^2} \quad \text{(a)} \end{aligned}$$

2. The volume of the tetrahedron formed by the points (1, 1, 1), (2, 1, 3), (3, 2, 2) and (3, 3, 4) in cubic units is

a) $\frac{5}{6}$

b) $\frac{6}{5}$

c) 5

d) $\frac{2}{3}$

$$\begin{aligned} \vec{AB} &= \hat{i} + 2\hat{k}, \quad \vec{AC} = 2\hat{i} + \hat{j} + \hat{k}, \quad \vec{AD} = 2\hat{i} + \hat{j} + 3\hat{k} \\ \text{Volume of the tetrahedron} &= \frac{1}{6} [\vec{AB} \times \vec{AC} \cdot \vec{AD}] = \frac{5}{6} \quad \text{(a)} \end{aligned}$$

3. Unit vector perpendicular to $\hat{i} - 2\hat{j} + 2\hat{k}$ and lying in the plane containing $\hat{i} + \hat{j} - 2\hat{k}$ and $-\hat{i} + 2\hat{j} + \hat{k}$ is

a) $8\hat{i} - 7\hat{j} + 11\hat{k}$

b) $8\hat{i} + 7\hat{j} - 11\hat{k}$

c) $8\hat{i} - 7\hat{j} - 11\hat{k}$

d) $\frac{1}{\sqrt{234}}(8\hat{i} - 7\hat{j} - 11\hat{k})$

only $\frac{1}{\sqrt{234}}(8\hat{i} - 7\hat{j} - 11\hat{k})$ is a unit vector and \perp to $\hat{i} - 2\hat{j} + 2\hat{k}$ (d)

Space for calculation / rough work
S.W.A.Y.



Mathematics

4. In the group $\mathcal{Q} = \{-1\}$ under the binary operation $*$ defined by $a * b = a + b + ab$ the inverse of 10 is

a) $\frac{1}{10}$

b) $\frac{11}{10}$

c) $\frac{-11}{10}$

d) $\frac{-10}{11}$

(d)

5. In the group $\{1, 2, 3, 4, 5, 6\}$ under multiplication mod 7, $2^{-1} \times 4 =$

a) 1

b) 4

c) 2

d) 3

(c)

6. The group $(\mathbb{Z}, +)$ has

a) exactly one subgroup

b) only two subgroups

c) no subgroups

d) infinitely many subgroups

(d) infinitely many subgroup.

Space for calculation / rough work

7. If $3x \equiv 5 \pmod{7}$, then

$3x \equiv 5 \pmod{7}$, then $x = 4$

a) $x \equiv 2 \pmod{7}$

Ans :-

$x \equiv 4 \pmod{7}$

b) $x \equiv 3 \pmod{7}$

(C)

c) $x \equiv 4 \pmod{7}$

d) none of these

8. The argument of the complex number $\sin\left(\frac{6\pi}{5}\right) + i\left(1 + \cos\frac{6\pi}{5}\right)$ is

a) $\frac{\pi}{10}$

(C)

b) $\frac{5\pi}{6}$

c) $\frac{-\pi}{10}$

d) $\frac{2\pi}{5}$

9. The maximum value of $n < 101$ such that $1 + \sum_{k=1}^n i^k = 0$ is

a) 96

(C)

b) 97

c) 99

d) 100

Space for calculation / rough work

Mathematics

16. The value of $(-1+\sqrt{-3})^{62} + (-1-\sqrt{-3})^{62}$ is

a) 2^{62}

b) 2^{61}

c) -2^{62}

d) 0

$$2^{62} \left[\left(\frac{-1+\sqrt{-3}}{2} \right)^{62} + \left(\frac{-1-\sqrt{-3}}{2} \right)^{62} \right]$$

$$2^{62} \{ \omega^{62} + \omega^{124} \} = 2^{62} \{ \omega^2 + \omega \} = 2^{62} (-1) = \underline{\underline{-2^{62}}}$$

17. All complex numbers z which satisfy the equation $\left| \frac{z-6i}{z+6i} \right| = 1$ lie on the

a) imaginary axis

b) real axis

c) neither of the axes

d) none of these

$$\left| \frac{(x+iy)-6i}{x+iy+6i} \right| = 1 \text{ or, } \frac{(x+iy)-6i}{x+iy+6i} \times \frac{x-iy-6i}{x-iy-6i} = 1$$

$$= \frac{(y^2+x^2+12y-36) + i12x}{x^2+(y+6)^2}$$

Solve it

12. The value of $\sin \left[\cot^{-1} \left\{ \cos \left(\tan^{-1} x \right) \right\} \right]$ is

a) $\left(\frac{1-x^2}{\sqrt{2-x^2}} \right)$

b) $\left(\frac{2+x^2}{\sqrt{1+x^2}} \right)$

c) $\left(\frac{\sqrt{x^2-2}}{\sqrt{x^2-1}} \right)$

d) $\left(\frac{x^2-1}{\sqrt{x^2-2}} \right)$

$$= \sin \left[\cot^{-1} \left\{ \cos \left(\cos^{-1} \frac{1}{\sqrt{1+x^2}} \right) \right\} \right]$$

$$= \sin \left[\cot^{-1} \left\{ \frac{1}{\sqrt{1+x^2}} \right\} \right]$$

$$= \underline{\underline{\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}}}$$

Space for calculation / rough work

13. The value of $\alpha (\neq 0)$ for which the function $f(x) = 1 + \alpha x$ is the inverse of itself is

- a) -2
- b) 2
- c) -1
- d) 1

(C)

Let $y = f(x)$, $y = 1 + \alpha x \Rightarrow x = \frac{y-1}{\alpha}$

$f(x)$ is the inverse of itself

$$\frac{x-1}{\alpha} = (1 + \alpha x)$$

$$\text{or, } (\alpha^2 - 1)x + (\alpha + 1) = 0$$

$$(\alpha + 1) \{ \alpha x - x + 1 \} = 0$$

14. If x^r occurs in the expansion of $(x + \frac{1}{x})^n$, then its coefficient is

$\alpha = -1$

- a) $\frac{n!}{(r!)^2}$
- b) $\frac{n!}{(r+1)!(r-1)!}$
- c) $\frac{n!}{(\frac{n+r}{2})! (\frac{n-r}{2})!}$
- d) $\frac{n!}{\left[\left(\frac{r}{2} \right)! \right]^2}$

(C)

k th term = ${}^n C_k x^k \left(\frac{1}{x} \right)^{n-k}$; coefficient of x^{2k-n}

Power of x ; x^{2k-n}

Let $x^{2k-n} = x^r$

$$r = 2k - n \Rightarrow k = \frac{n+r}{2}$$

Now ${}^n C_k = {}^n C_{\frac{n+r}{2}} = \frac{n!}{\frac{n+r}{2}! \cdot \frac{n-r}{2}!}$

15. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$ then $\cot(A-B) =$

- a) $\frac{1}{y} - \frac{1}{x}$
- b) $\frac{1}{x} - \frac{1}{y}$
- c) $\frac{1}{x} + \frac{1}{y}$
- d) none of these

(C)

$$\cot(A-B) = \frac{1 + \tan A \cdot \tan B}{\tan A - \tan B} \quad \text{--- (1)}$$

$$\frac{1}{\tan B} - \frac{1}{\tan A} = \frac{\tan A - \tan B}{\tan A \cdot \tan B} = y \quad \text{--- (2) Given: } \tan - \tan B = x \quad \text{--- (3)}$$

Eq (2) + (3) \div $\tan A \cdot \tan B = x/y$; put this value in eq (1)

$$\cot(A-B) = \frac{1 + x/y}{x} = \frac{1}{x} + \frac{1}{y}$$

Space for calculation / rough work

✓ $\cos^2 \frac{\pi}{12} \cos^2 \frac{\pi}{4} \cos^2 \frac{5\pi}{12} = \cos^2 15^\circ + \frac{1}{2} + \cos^2 75^\circ$

✓ $\frac{3}{2}$

b) $\frac{3-\sqrt{3}}{2}$

c) $\frac{2}{3}$

d) $\frac{2}{3+\sqrt{3}}$

(a)

$= \cos^2 15^\circ + 1 - \sin^2 75^\circ + \frac{1}{2}$

$= \cos^2 15^\circ - \sin^2 75^\circ + \frac{3}{2}$

$= \cos^2 15^\circ - \cos^2 (90^\circ - 15^\circ) + \frac{3}{2}$

$= \frac{3}{2} + \cos^2 15^\circ - \cos^2 15^\circ$

$= \frac{3}{2}$

19. If

- a)
- b)
- c)
- d)

20. If

- a)
- b)
- c)
- d)

21. If

- a)
- b)
- c)
- d)

17. If $\sin \theta$, $\cos \theta$, and $\tan \theta$ are in GP then $\cot^2 \theta - \cot^4 \theta$ is

✓ a) 1

b) $\frac{1}{2}$

c) 2

d) 3

(a)

$\cos^2 \theta = \sin \theta \cdot \tan \theta \Rightarrow \cos^3 \theta = \sin^2 \theta$

or, $\cos^3 \theta = 1 - \cos^2 \theta$ or, $\cos^3 \theta + \cos^2 \theta - 1 = 0$

Solve it for θ and replace in $\cot^2 \theta - \cot^4 \theta = \underline{1}$

18. If $\frac{3x^2 - 2x + 4}{(x-1)^2} = \frac{A_1}{x+1} + \frac{A_2}{(x+1)^2} + \frac{A_3}{(x+1)^3} + \frac{A_4}{(x+1)^4} + \frac{A_5}{(x+1)^5} + \frac{A_6}{(x+1)^6}$, then

$(-A_1 + A_2 + A_3, A_2 - A_4 - A_6) =$

a) (0, 0)

b) (-8, -12)

c) (8, -12)

✓ d) (-8, 12)

(d)

Put $x=0$,

then $A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 4$

only "d" (-8, 12) satisfies

the solution

22. The

- a)
- b)
- c)
- d)

Space for calculation / rough work

19. If $\log_2(2^{x-1}+6) + \log_2(4^{x-1}) = 5$, then $x =$; $\log_2(2^{x-1}+6)(2^{2x-2}) = 5$

- a) 4
- b) 1
- c) 3
- d) 2

(d)

or, $(2^{x-1}+6)(2^{2x-2}) = 2^5$; Let $y = 2^{x-1}$
 Possible $(y+6)y^2 = 32$ or $(y-2)(y^2+8y+16) = 0$
 soln $y = 2^{x-1} = 2^1 \therefore x-1 = 1$ or, $x = 2$

20. If a, b, c, d are the roots of the equation $x^4 + 2x^3 + 3x^2 + 4x + 5 = 0$, then $1+a^2+b^2+c^2+d^2$ is equal to

- a) -2
- b) -1
- c) 2
- d) 1

(b)

$1+(a^2+b^2+c^2+d^2) = 1+(a+b+c+d)^2 - 2(ab+ac+ad+bc+bd+cd)$
 $= 1 + (\text{sum of roots})^2 - 2(\text{sum of multiplication of roots})$
 $= 1 + 2^2 - 2 \times 3 = 5 - 6 = -1$ Ans

21. If $C_0, C_1, C_2, \dots, C_n$ are binomial coefficients of order n , then the value of $\frac{C_1}{2} + \frac{C_2}{4} + \frac{C_3}{6} + \dots =$

- a) $\frac{2^n+1}{n+1}$
- b) $\frac{2^n-1}{n+1}$
- c) $\frac{2^n+1}{n-1}$
- d) $\frac{2^n}{n+1}$

(b)

$(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$
 Integrate both side from 0 to 1:
 $\frac{2^n-1}{n+1} = C_0 + C_1/2 + C_2/3 + \dots + C_n/n+1$ — (1)
 again $(1-x)^n = C_0 - C_1x + C_2x^2 + \dots + (-1)^n C_nx^n$
 Integrate both side 0 to 1 (\div)
 $\frac{1}{n+1} = C_0 - C_1/2 + C_2/3 - \dots$ — (2) Perform (1) - (2)
 result $\Rightarrow \frac{2^n-1}{n+1} = C_1/2 + \frac{C_2}{4} + \frac{C_3}{6} + \dots$

22. The value of $(0.2)^{\log_{\sqrt{5}}(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \infty)}$ is

- a) 4
- b) $\frac{1}{4}$
- c) 2
- d) $\frac{1}{2}$

(a)

$(0.2)^{\log_{\sqrt{5}}(\frac{1}{4} + \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4}(\frac{1}{2})^2 + \dots + \infty)}$
 $= (0.2)^{\log_{\sqrt{5}}(\frac{\sqrt{4}}{1-1/2})} = (0.2)^{\log_{\sqrt{5}}(4/2)}$
 $= (0.2)^{-1 \log_{\sqrt{5}} 2}$
 $= (\frac{1}{5})^{-\log_{\sqrt{5}} 2} = 5^{\log_{\sqrt{5}} 2}$
 $= 5 \log_5 4 = 4$

Space for calculation / rough work

27. If $n(A) = n(B) = m$, then the number of possible bijections from A to B is

a) m

b) m^m

c) $m!$

d) $2m$

(C) $m!$

28. $\sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}] =$

a) $\sin^{-1} x - \sin^{-1} \sqrt{1-x}$

b) $\sin^{-1} x + \sin^{-1} \sqrt{1-x}$

c) $\sin^{-1} x - \sin^{-1} \sqrt{x}$

d) $\sin^{-1} x + \sin^{-1} \sqrt{x}$

(C)

$$\begin{aligned} & \sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}] \\ &= \sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}] \quad \text{--- (1)} \\ &= \sin^{-1} [x\sqrt{1-(\sqrt{x})^2} - \sqrt{x}\sqrt{1-x^2}] \quad \text{--- (1)} \end{aligned}$$

w.k.t.

$$\sin^{-1}(x+y) = \sin^{-1} [x\sqrt{1-y^2} - y\sqrt{1-x^2}] \quad \text{--- (2)}$$

compare (1) & (2) :

$$\boxed{\sin^{-1} x - \sin^{-1} \sqrt{x}}$$

29. If $\tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \tan 4\theta \tan 7\theta$, then the general solution is

a) $\theta = \frac{n\pi}{4}$

b) $\theta = \frac{n\pi}{12}$

c) $\theta = \frac{n\pi}{6}$

d) none of these

(b)

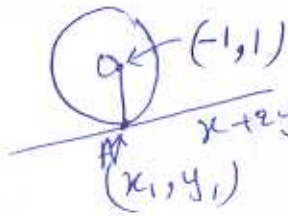
30. If a circle with the point $(-1, 1)$ as its center touches the straight line $x+2y+9=0$ then the coordinates of the points of contact is

a) $(-3, 3)$

b) $(-3, -3)$

c) $(0, 0)$

d) $(\frac{7}{3}, -\frac{17}{3})$



1st eqn of $x+2y+9=0$ is

$$y = -\frac{x}{2} - \frac{9}{2} \quad \text{--- (1)}$$

Eqn of OA $\Rightarrow 1 = -2 + c$

$$c = -3$$

$$y = -\frac{x}{2} - 3 \quad \text{--- (2)}$$

Solve eqn (1) & (2) :

Space for calculation/rough work.

$$x = -3$$

$$y = -3$$

Ans we r - **(b)**

27. If the circles $x^2 + y^2 + 2gx + 2fy = 0$, and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other, then

For given condition:

$$\frac{2g}{2g'} = \frac{2f}{2f'} \Rightarrow \boxed{f'g = g'f}$$

- a) $fg = f'g'$ (b)
- b) $f'g = fg'$
- c) $ff' = gg'$
- d) none of these

28. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 4x + 2y - 4 = 0$ is

- a) 1
 - b) 2
 - c) 3
 - d) 4
- Eqns are: $x^2 + y^2 = (2)^2$; (1) Cut each other at two places: So, no. common tangents = 2
- (b) $(x-2)^2 + (y+1)^2 = 3^2$; (2)

29. The length of the tangent drawn from any point on the circle $x^2 + y^2 - 4x + 6y - 4 = 0$ to the circle $x^2 + y^2 - 4x + 6y = 0$ is

- a) 8
 - b) 4
 - c) 2
 - d) none of these
- (c) Length of tangent from $x^2 + y^2 + 2gx + 2fy + c_1 = 0$ to circle $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ is $\sqrt{c_2 - c_1}$ here $c_2 = 4, c_1 = 0$

So, length = $\sqrt{4-0} = 2$

30. If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is

- a) 25
 - b) 9
 - c) 16
 - d) 4
- For hyperbola = $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$; eccentricity $e = \frac{225}{144} = \frac{15}{12}$
 foci $(\pm \frac{12}{5} \times \frac{15}{12}, 0) = (\pm 3, 0)$
 Since foci of ellipse coincide: $\Rightarrow 5e = 3$ or, $e = 3/5$
 since $b^2 = a^2(1 - e^2)$ or $b^2 = 25(1 - 9/25) = 16$ ✓

31. The latus rectum of the ellipse is half the minor axis. Then its eccentricity is

- a) $\frac{1}{\sqrt{2}}$
 - b) $\frac{1}{\sqrt{3}}$
 - c) $\frac{\sqrt{3}}{2}$
 - d) none of these
- (c) Latus Rectum = $2b^2/a$, L minor axis = $2b$
 Given $\frac{2b^2}{2} = 2b^2/a \Rightarrow a = 2b$
 W.K.T. $b^2 = a^2(1 - e^2)$
 $1 - e^2 = 1/4$ or, $e^2 = 3/4$
 $\boxed{e = \sqrt{3}/2}$

Space for calculation/work

Mathematics

32. The ends of the latus rectum of the parabola $x^2 + 10x - 16y + 25 = 0$ are

- a) (3,4), (-13,4)
- b) (5,-8), (-5,8)
- c) (3,-4), (13,4)
- d) (-3,-4), (13,-4)

(a)

$$(x+5)^2 = 4(4)y$$

$$x^2 = 4ay$$

vertex = (-5,0), focus = (-5,4)

eqn of axis $\Rightarrow x = 4$

only (3,4) eqn of a line \perp to axis and passing through focus is $y = 4$; (3,4), (-13,4)

Substituting the parabola and their y-coordinates is 4

36. If

a)

33. Which of the following functions is differentiable at $x=0$?

- a) $\cos(|x|) + |x|$
- b) $\cos(|x|) - |x|$
- c) $\sin(|x|) + |x|$
- d) $\sin(|x|) - |x|$

(d)

d)

34. If $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$, $y = a \sin t$, then $\frac{dy}{dx} =$

- a) $\tan t$
- b) $\cot t$
- c) $-\cot t$
- d) $-\tan t$

(a)

Find $\frac{dy}{dt}$ and $\frac{dx}{dt}$

$$\frac{(dy/dt)}{(dx/dt)} = \frac{\cos t \sin t}{\cos^2 t} = \tan t$$

37. If

a)

b)

c)

d)

35. If $\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

$$\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \frac{1}{\sec^2 \theta}$$

a) $a=1, b=1$

b) $a = \cos 2\theta, b = \sin 2\theta$

c) $a = \sin 2\theta, b = \cos 2\theta$

d) $a = \cos \theta, b = \sin \theta$

(b)

$$= \begin{bmatrix} 1 - \tan^2 \theta & -2 \tan \theta \\ 2 \tan \theta & 1 - \tan^2 \theta \end{bmatrix} \frac{1}{\sec^2 \theta}$$

$$= \begin{bmatrix} \cos^2 \theta - \sin^2 \theta & -2 \sin \theta \cos \theta \\ 2 \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix}$$

Space for calculation / rough work

Answers
 1-10
 11-20
 21-30
 31-40

$$\begin{pmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{pmatrix}^{-1} = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$a = \cos 2\theta$
 $b = \sin 2\theta$

Slu...

36. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^n is

a) $\begin{bmatrix} 1 & 2^n - 2 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & n^2 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

37. If α, β, γ are the roots of the equation $x^3 + px + q = 0$ then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

a) q

b) 0

c) p

d) $p^2 - 2q$

(b)

38. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$ in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is

a) 0

b) 1

c) 2

d) 3

(b)

Space for calculation / rough work

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Mathematics

Ver A Mat

39. The sum of non-prime positive divisors of 450 is
 a) 1209
 b) 1299
 ✓ c) 1199
 d) 1099

(c)

40. The last digit of $\sum_{\substack{1 < p < 100 \\ p \text{ - prime}}} p! - \sum_{n=1}^{50} (2n)!$ is

- a) 2
 b) 4
 c) 6
 ✓ d) 8

(d)

41. The interval I such that $\int_0^1 \frac{dx}{\sqrt{1+x^4}} \in I$ is given by

- a) $(0, \frac{1}{\sqrt{2}})$
 ✓ b) $[\frac{1}{\sqrt{2}}, 1]$
 c) $[\sqrt{2}, 2]$
 d) $[\sqrt{2}, \frac{7}{4}]$

$(1+x^4) < (1+x^2)^2 \Rightarrow \sqrt{1+x^4} < 1+x^2$
 $\therefore \frac{1}{\sqrt{1+x^4}} > \frac{1}{1+x^2} \quad \therefore \frac{1}{1+x^2} < \frac{1}{\sqrt{1+x^4}}$
 $\frac{1}{\sqrt{1+x^4}} < 1$ always.
 $\therefore \int_0^1 \frac{1}{1+x^2} dx < \int_0^1 \frac{1}{\sqrt{1+x^4}} dx < \int_0^1 1 dx$

(b)

42. $\int_0^{\frac{\pi}{2}} \log(\tan x) dx =$

- a) $\frac{\pi}{2}$
 ✓ b) 0
 c) 1
 d) $\frac{\pi}{4}$

(b)

$\int_0^{\frac{\pi}{2}} \log(\tan x) dx = \int_0^{\frac{\pi}{2}} \log \sin x - \int_0^{\frac{\pi}{2}} \log \cos x$
 $= 0$
 $\Rightarrow \int_0^{\frac{\pi}{2}} \log(\sin x) = \int_0^{\frac{\pi}{2}} \log(\cos x)$

Space for calculation / rough work

43. The value of $\int_{-2}^2 (ax^3 + bx + c) dx$ depends on the

- a) value of b
- b) value of c
- c) value of a
- d) values of a and b

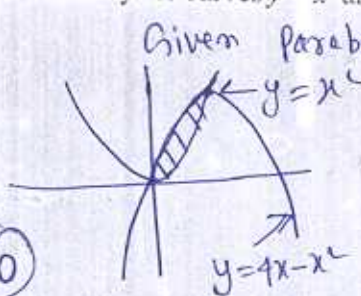
(b)

Since $ax^3 + bx$ is an odd function $\int_{-2}^2 (ax^3 + bx) dx = 0$
 Hence $\int_{-2}^2 (ax^3 + bx + c) dx = \int_{-2}^2 c dx$; \therefore integral depends upon the value of 'c'

44. The area of the region bound by the curves $y = x^2$ and $y = 4x - x^2$ is

- a) $\frac{16}{3}$ sq. units
- b) $\frac{8}{3}$ sq. units
- c) $\frac{4}{3}$ sq. units
- d) $\frac{2}{3}$ sq. units

(b)



Given parabolas are $y = x^2$, $(y-4) = -(x-2)^2$

x-coordinates on intersect pt. = 0 or 2

$$\text{area} = \int_0^2 (4x - x^2 - x^2) dx = \int_0^2 (4x - 2x^2) dx$$

$$\left[2x^2 - \frac{2}{3}x^3 \right]_0^2 = \underline{\underline{8/3}}$$

45. The particular solution of $\frac{y}{x} \frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, when $x=1, y=2$ is

- a) $5(1+y^2) = 2(1+x^2)$
- b) $2(1+y^2) = 5(1+x^2)$
- c) $5(1+y^2) = (1+x^2)$
- d) $(1+y^2) = 2(1+x^2)$

(b)

$$\frac{1}{2} \int \frac{2y dy}{1+y^2} = \frac{1}{2} \int \frac{2x dx}{1+x^2}; \text{ or, } \frac{1}{2} \log(1+y^2)$$

$$= \frac{1}{2} \log(1+x^2) + c$$

or, $\log\left(\frac{1+y^2}{1+x^2}\right) = c$
 Put $x=1, y=2$ Put value of c in eqn
 $c = \log 5/5$ $\therefore 2(1+y^2) = 5(1+x^2)$

46. The solution of the differential equation $\frac{dy}{dx} = (x+y)^2$ is

- a) $\frac{1}{x+y} = c$
- b) $\sin^{-1}(x+y) = x + c$
- c) $\tan^{-1}(x+y) = c$
- d) $\tan^{-1}(x+y) = x + c$

(d)

Put $x+y = z \Rightarrow \frac{dy}{dx} + 1 = \frac{dz}{dx}$

Now given eqn

$$\frac{dz}{dx} - 1 = z^2 \text{ or, } \frac{dz}{dx} = 1 + z^2$$

$$\int dx = \int \frac{dz}{1+z^2}$$

Space for calculation / rough work

$c + x = \tan^{-1}(x+y)$

47. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is

- 1. $\int e^x$
- a)
- b)
- c)
- d)

- a) $x^{1/2}$
- b) \sqrt{e}
- c) 1
- d) e^2

48. Let x be a number which exceeds its square by the greatest possible quantity, then $x =$

- a)
- b)
- c)
- d)

- a) $1/2$
- b) $1/4$
- c) $3/4$
- d) $1/3$

Go by option $\frac{1}{2}$ For $x = \frac{1}{2}$
 $\frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

49. The subtangent at $x = \pi/2$ on the curve $y = x \sin x$ is

- 2. $\int \frac{x}{x}$
- a)
- b)
- c)
- d)

- a) 0
- b) 1
- c) $\pi/2$
- d) none of these

Slope $\frac{dy}{dx} \Big|_{x=\pi/2} = 1$; $y = x + c$; $P(\pi/2, 0)$ lies on $y = x \sin x$; so $c = \pi/2$
 eqn of line $\Rightarrow y = x + \pi/2$ (distance b/w origin and axis intersect Pt = $\pi/2$)

50. The value of $\int \frac{10^{x/2}}{\sqrt{10^{-x} - 10^x}} dx$ is

- a)
- b)
- c)
- d)

- a) $\frac{1}{\log_e 10} \sin^{-1}(10^x) + c$
- b) $2\sqrt{10^{-x} + 10^x} + c$
- c) $\frac{1}{\log_e 10} \sinh^{-1}(10^x) + c$
- d) $\frac{-1}{\log_e 10} \sinh^{-1}(10^x) + c$

$$\int \frac{10^{x/2} dx}{\sqrt{10^{-x} - 10^x}} = \int \frac{10^{x/2} 10^{x/2} dx}{\sqrt{1 - (10^x)^2}}$$

$$= \int \frac{10^x dx}{\sqrt{1 - (10^x)^2}}; y = 10^x = e^{x \log_e 10}$$

$$\frac{dy}{dx} = (\log_e 10) e^{x \log_e 10}$$

$$= \int \frac{\log_e 10 (e^{x \log_e 10}) dx}{\log_e 10 \sqrt{1 - (10^x)^2}}$$

$$= \frac{1}{\log_e 10} \int \frac{dy}{\sqrt{1 - y^2}}; y = 10^x = e^{x \log_e 10}$$

$$= \frac{1}{\log_e 10} \sin^{-1}(10^x) + c$$

Sp for calculation / rough work

11. $\int e^x \left\{ \frac{1 + \sin x \cos x}{\cos^2 x} \right\} dx =$

$\int e^x \left\{ \frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2} \right\} dx$

- a) $e^x \cos x + c$
- b) $e^x \sec x \tan x + c$
- c) $e^x \tan x + c$
- d) $e^x \cos^2 x - 1 + c$

(c) $\int e^x (\sec^2 x + \tan x) dx \equiv \int e^x (f'(x) + f(x)) dx$
 $= e^x \tan x + c$

12. $\int \frac{x^2 + 1}{x^4 + 1} dx$

(d)

- a) $\frac{1}{\sqrt{2}} \log_e(x^2 + 1) + c$
- b) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 + 1}{x\sqrt{2}} \right) + c$
- c) $\frac{1}{\sqrt{2}} \tan^{-1}(x^2 - 1) + c$
- d) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{x\sqrt{2}} \right) + c$

13. The locus of the mid point of the intercept of the line $x \cos \alpha + y \sin \alpha = p$ between the coordinate axes is

- a) $x^2 + y^2 = 4p^2$
- b) $x^2 + y^2 = p^2$
- c) $x^2 + y^2 = 4p^{-2}$
- d) $x^2 + y^2 = p^2$

(a) when $x = 0, y = p \operatorname{cosec} \alpha$
 $y = 0, x = p \operatorname{sec} \alpha$
 mid point $\equiv \left(\frac{p \operatorname{sec} \alpha}{2}, \frac{p \operatorname{cosec} \alpha}{2} \right)$
 $\therefore x = \frac{p \operatorname{sec} \alpha}{2}; y = \frac{p \operatorname{cosec} \alpha}{2}$
 $\therefore \cos \alpha = \frac{p}{2x}; \sin \alpha = \frac{p}{2y}$
 W.K.T $\cos^2 \alpha + \sin^2 \alpha = 1$

Space for calculation / rough work

$\frac{p^2}{4x^2} + \frac{p^2}{4y^2} = 1$
 $x^2 + y^2 = 4p^2$

Mathematics

Ver A Mathe

54. If the line through $A = (4, -5)$ is inclined at an angle 45° with the positive direction of the x-axis, then the coordinates of the two points on opposite sides of A at a distance of $3\sqrt{2}$ units are

- a) (7, 2), (1, 8)
- b) (7, 2), (1, -8)
- c) (7, -2), (1, -8)
- d) (7, 2), (-1, 8)

Slope = $\tan 45^\circ = 1$; eqn $y = x + C$
 $P(4, 5)$ lies on line so, $C = -9$
 Now, eqn $\Rightarrow y = x - 9$
 only (7, -2) and (1, -8) lies on above st. line
 eqn, do no need for further calculation

- 57. lim
- a)
- b)
- c)
- d)

55. If the line $px + qy = 0$ coincides with one of the lines given by $ax^2 + 2hxy + by^2 = 0$ then

- a) $ap^2 + 2hpq + bq^2 = 0$
- b) $aq^2 + 2hpq + bp^2 = 0$
- c) $aq^2 - 2hpq + bp^2 = 0$
- d) none of these

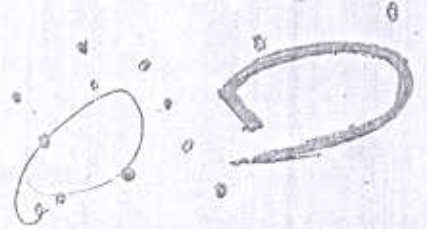
$y = \frac{-p}{q}x$; put in given pair of lines
 or, $ax^2 + 2hx(\frac{-px}{q}) + b\frac{p^2}{q^2}x^2 = 0$
 or, $(aq^2 - 2hpq + bp^2)x^2 = 0$
 Soln: either $x = 0$ or $aq^2 - 2hpq + bp^2 = 0$

- 58. The n
- a) 3
- b) 2
- c) 1
- d) 6

56. The function $f(x) = \left(\frac{\log_e(1+ax) - \log_e(1-bx)}{x} \right)$ is undefined at $x = 0$. The value which should be assigned to f at $x = 0$ so that it is continuous at $x = 0$ is

- a) $\frac{a+b}{2}$
- b) $a+b$
- c) $\log_e(ab)$
- d) $a-b$

(b)



- 59. The a
- a) 17
- b) 14
- c) 13
- d) 12

Space for calculation / rough work



67. $\lim_{n \rightarrow \infty} \frac{(1^2 + 2^2 + \dots + n^2) \sqrt{n}}{(n+1)(n+10)(n+100)} =$

- a) 3
- b) $\frac{1}{3}$
- c) $\frac{2}{3}$
- d) ∞

(b)

$$\left(\lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6(n+1)(n+10)(n+100)} \right) \left(\lim_{n \rightarrow \infty} \sqrt{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2n^2 + n}{6(n+10)(n+100)} (1) = \lim_{n \rightarrow \infty} \frac{2 + 1/n}{6(1+10/n)(1+100/n)}$$

$$= \frac{2+0}{6(1+0)(1+0)} = \frac{2}{6} = \frac{1}{3}$$

58. The number of triangles in a complete graph with 10 non-collinear vertices is

- a) 360
- b) 240
- c) 120
- d) 60

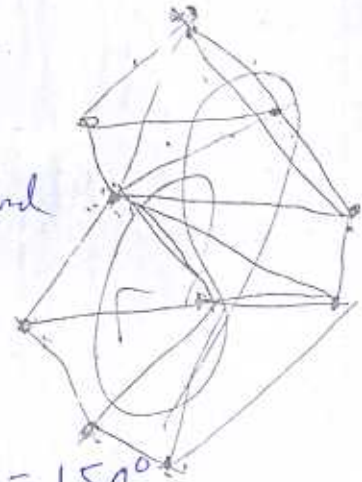
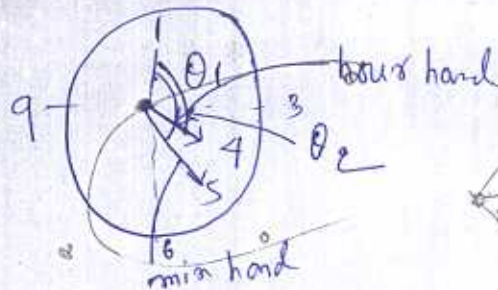
(c)

no. of triangle = ${}^{10}C_3 = \frac{10 \times 9 \times 8 \times 7}{2 \times 3} = 120$

59. The angle between hands of a clock when the time is 4.25 AM is

- a) $17 \frac{1}{2}^\circ$
- b) $14 \frac{1}{2}^\circ$
- c) $13 \frac{1}{2}^\circ$
- d) $12 \frac{1}{2}^\circ$

(a)



θ_1 min hand = $\frac{360^\circ}{12} \times 5 = 150^\circ$

θ_2 hour hand = $\frac{360^\circ}{12} \times 4 + \frac{30^\circ}{60 \text{ min}} \times 25 \text{ min}$

$\theta_1 - \theta_2 = 150 - 132.5 = 17.5^\circ = 17 \frac{1}{2}^\circ$

Space for calculation / rough work

