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Total No. of Questions : 24
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Part-III

MATHEMATICS, Paper - I (A)

(English version)

Time : 3 Hours]

[Max. Marks : 75

Note : This question paper consists of **three** sections **A, B** and **C**.

SECTION - A

10×2=20

I. Very short answer type questions.

(i) Attempt **all** the questions.

(ii) Each question carries **TWO** marks.

1. If $f: \mathbb{Q} \rightarrow \mathbb{Q}$ defined by $f(x) = 5x + 4$, then find f^{-1} .

2. Find the domain of the real function $f(x) = \sqrt{4x - x^2}$.

3. If $\vec{a} = 2\vec{i} + 4\vec{j} - 5\vec{k}$, $\vec{b} = \vec{i} + \vec{j} + \vec{k}$, $\vec{c} = \vec{j} + 2\vec{k}$, then find a unit vector in the opposite direction of $\vec{a} + \vec{b} + \vec{c}$.

4. If the vectors $-3\vec{i} + 4\vec{j} + \lambda\vec{k}$ and $\mu\vec{i} + 8\vec{j} + 6\vec{k}$ are collinear vectors, then find λ and μ .

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5. Find the area of the parallelogram $2\vec{i} - 3\vec{j}$ and $3\vec{i} - \vec{k}$ as adjacent sides.
6. If $\sec \theta = x + \frac{1}{4x}$, prove that $\tan \theta + \sec \theta = 2x$.
7. Show that $\cos 100^\circ \cos 40^\circ + \sin 100^\circ \sin 40^\circ = \frac{1}{2}$.
8. If $\sinh x = \frac{1}{2}$, find the value of $\cosh 2x + \sinh 2x$.
9. In ΔABC , $a = 4$, $b = 5$, $c = 7$, then find the value of $\cos\left(\frac{B}{2}\right)$.
10. Find the square root of $3 + 4i$.

SECTION - B

5×4=20

II. Short answer type questions.

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors, then prove that the four points $-\vec{a} + 4\vec{b} - 3\vec{c}$, $3\vec{a} + 2\vec{b} - 5\vec{c}$, $-3\vec{a} + 8\vec{b} - 5\vec{c}$ and $-3\vec{a} + 2\vec{b} + \vec{c}$ are coplanar.

12. Prove by vector method, the angle between the two diagonals of a Cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

13. If A is not an integral multiple of π , then prove that

$$\cos A \cdot \cos 2A \cdot \cos 4A \cdot \cos 8A = \frac{\sin 16A}{16 \sin A}$$

14. Solve $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$.
15. Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{36}{85}\right)$.
16. Prove that $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$
17. Show that $2^5 \cos^2 \theta \sin^4 \theta = \cos 6\theta - 2 \cos 4\theta - \cos 2\theta + 2$

SECTION - C

5×7=35

III. Long answer type questions.

- (i) Attempt **ANY FIVE** questions.
- (ii) Each question carries **SEVEN** marks.

18. If $f: A \rightarrow B$, $g: B \rightarrow C$ are bijections, then prove that $g \circ f: A \rightarrow C$ is a bijection.

19. By using Mathematical induction, show that $3 \cdot 5^{2n+1} + 2^{3n+1}$ is divisible by 17. $n \in \mathbb{N}$.

20. Let $\vec{a} = i + j + k$, $\vec{b} = 2i - j + 3k$, $\vec{c} = i - j$ and $\vec{d} = 6i + 2j + 3k$.

Express \vec{d} in terms of $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ and $\vec{a} \times \vec{b}$.

21. If A, B, C are angles of a triangle, then prove that

$$\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}.$$

22. If $a = 13$, $b = 14$, $c = 15$, then show that

$$R = \frac{65}{8}, \quad r = 4, \quad r_1 = \frac{21}{2}, \quad r_2 = 12 \quad \text{and} \quad r_3 = 14.$$

23. On a tower AB of height 'h', there is a flag-staff BC at a point of d metres away from the foot of the tower. AB and BC are making equal angles. Show

that the height of the flag - staff is $h \left[\frac{d^2 + h^2}{d^2 - h^2} \right]$ metres.

24. Show that one value of $\left[\frac{1 + \sin \frac{\pi}{8} + i \cos \frac{\pi}{8}}{1 + \sin \frac{\pi}{8} - i \cos \frac{\pi}{8}} \right]^{\frac{8}{3}} = -1$