IIT-JEE 2012

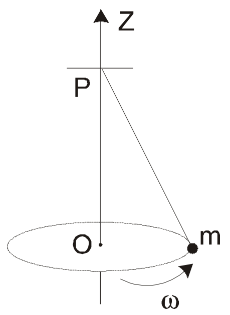
PAPER - 1

PART - I : PHYSICS

Section I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion is the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time.

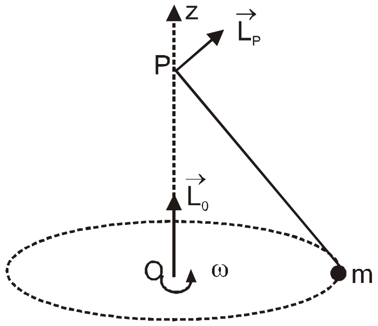
(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.

(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

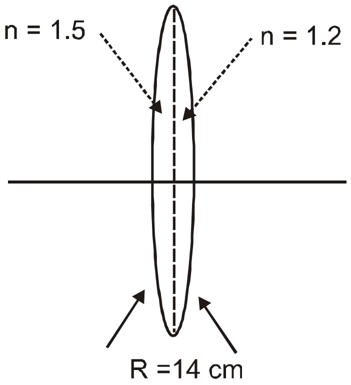
Ans. **(C)**

Sol.



magnitude and direction of L_0 remain constant
Magnitude of L_P remains constant but direction of L_P changes.

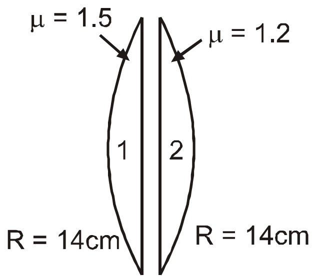
2. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(A) – 280.0 cm. (B) 40.0 cm. (C) 21.5 cm. (D) 13.3 cm.

Ans. (B)

Sol.



$$\frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f_1} = (1.5 - 1)\left[\frac{1}{14} - \frac{1}{\infty}\right]$$

$$\frac{1}{f_1} = \frac{0.5}{14}$$

$$\frac{1}{f_1} = (1.2 - 1)\left[\frac{1}{\infty} - \frac{1}{-14}\right]$$

$$\frac{1}{f_2} = \frac{0.2}{14}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{0.5}{14} + \frac{0.2}{14}$$

$$\frac{1}{f} = \frac{0.7}{14}$$

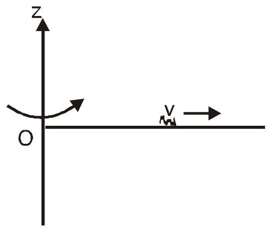
$$\frac{1}{v} = \frac{7}{140} - \frac{1}{40}$$

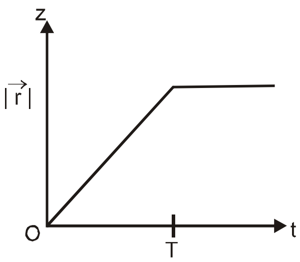
$$= \frac{1}{20} - \frac{1}{40}$$

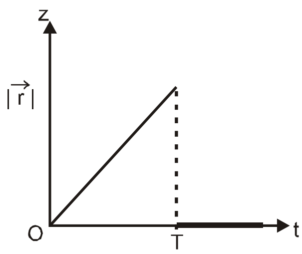
$$\frac{1}{v} = \frac{2-1}{40}$$

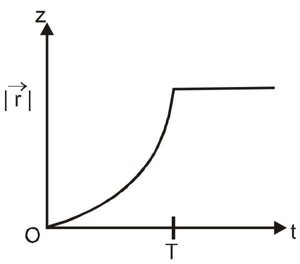
v = 40 cm

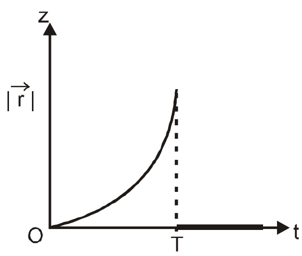
3. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time, t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) on the system about O, as a function of time is best represented by which plot ?



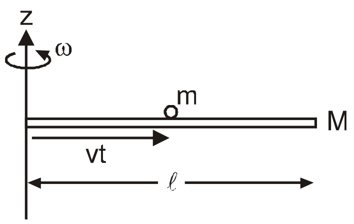
(A)


(B)


(C)


(D)


Ans. **(B)**

Sol.


$L = [m(vt)^2]\omega$

$L = mv^2\omega t^2$

So $\tau = \dfrac{dL}{dt} = 2mv^2\omega t$

$\tau \propto t$

$\Rightarrow$ straight line passing through (0, 0)

4. A mixture of 2 moles of helium gas (atomic mass = 4 amu), and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\dfrac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is :

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

Ans. **(D)**

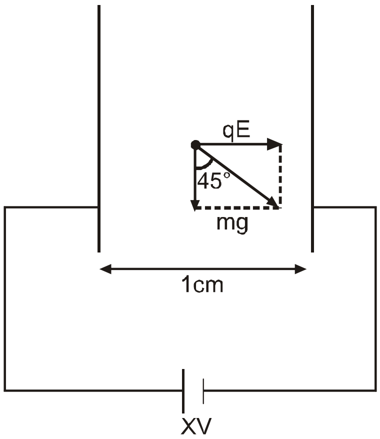
Sol. $\dfrac{v_{Rms_{He}}}{v_{Rms_{Ar}}} = \dfrac{\sqrt{\dfrac{3RT}{m_{He}}}}{\sqrt{\dfrac{3RT}{m_{Ar}}}} = \sqrt{\dfrac{m_{Ar}}{m_{He}}} = \sqrt{\dfrac{40}{4}} = \sqrt{10} \approx 3.16$

5. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly

(A) 1×10^{-5} V (B) 1×10^{-7} V (C) 1×10^{-9} V (D) 1×10^{-10} V

Ans. **(A)**

Sol.



mg = qE

$1.67 \times 10^{-27} \times 10 = 1.6 \times 10^{-19} \times \dfrac{X}{0.01}$

$X = \dfrac{1.67}{1.6} \times 10^{-9}$ V

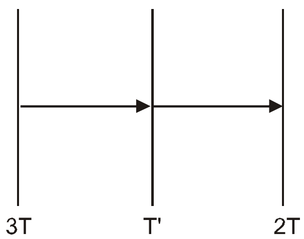
$X = 1 \times 10^{-9}$ V

6. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(A) $\left(\dfrac{65}{2}\right)^{\frac{1}{4}} T$ (B) $\left(\dfrac{97}{4}\right)^{\frac{1}{4}} T$ (C) $\left(\dfrac{97}{2}\right)^{\frac{1}{4}} T$ (D) $(97)^{\frac{1}{4}} T$

Ans. **(C)**

Sol.



In steady state energy absorbed by middle plate is equal to energy released by middle plate.

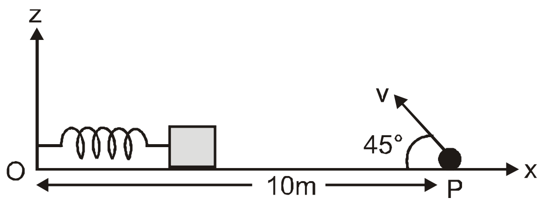
$\sigma A(3T)^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T)^4$

$(3T)^4 - (T')^4 = (T')^4 - (2T)^4$

$(2T')^4 = (16 + 81)\, T^4$

$T' = \left(\dfrac{97}{2}\right)^{1/4} T$

7. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3} \text{rad/s}$. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 cm from O. If the pebble hits the block at t = 1s, the value of v is (take g = 10 m/s^2)



(A) $\sqrt{50}\,\text{m/s}$ (B) $\sqrt{51}\,\text{m/s}$ (C) $\sqrt{52}\,\text{m/s}$ (D) $\sqrt{53}\,\text{m/s}$

Ans. (A)

Sol. Time of flight for projectile

$$T = \frac{2u\sin\theta}{g} = 1 \text{ sec.}$$

$$\frac{2u\sin 45}{g} = 1 \text{ sec.}$$

$$u = \frac{g}{\sqrt{2}}$$

$$u = \sqrt{50} \text{ m/s}$$

8. Young's double slit experiment is carried out by using green, red and blue light, one color at time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$ (C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

Ans. (D)

Sol. $\beta = \frac{\lambda D}{d}$

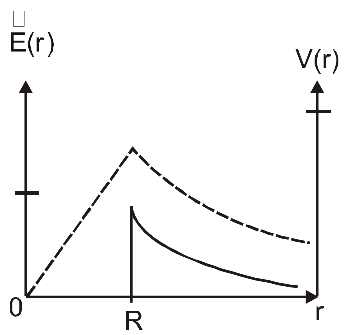
$\overrightarrow{\text{VIBGYOR}}$ λ increase

$\lambda_R > \lambda_G > \lambda_B$

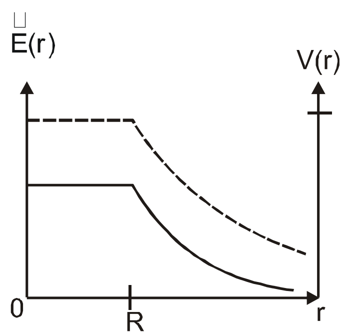
So $\beta_R > \beta_G > \beta_B$

9. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

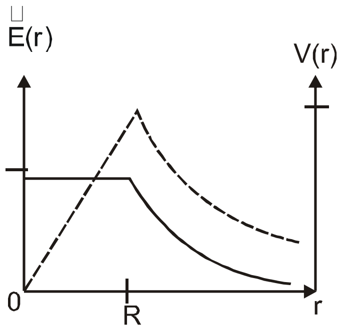
(A)



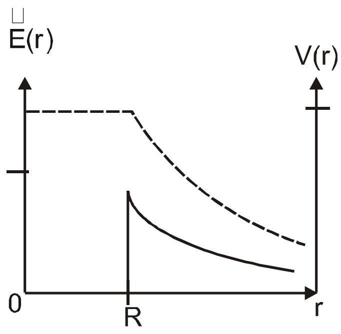
(B)



(C)

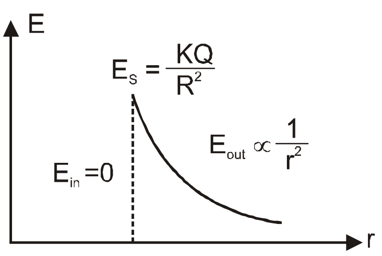


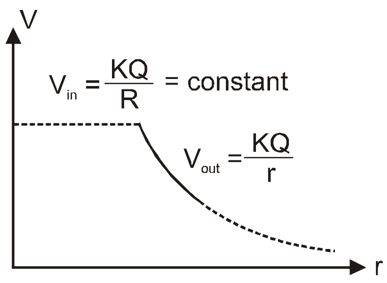
(D)



Ans. **(D)**

Sol.





10. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension ℓ = 0.25 mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(A) due to the errors in the measurements of d and ℓ are the same.

(B) due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.

(C) due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.

(D) due to the error in the measurement of d is four time that due to the error in the measurement of ℓ.

Ans. **(A)**

Sol. $\Delta d = \Delta \ell = \frac{0.5}{100}$ mm

$$y = \frac{4MLg}{\pi \ell d^2}$$

$$\left(\frac{\Delta y}{y}\right)_{max} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta d}{d}$$

error due to ℓ measurement $\frac{\Delta \ell}{\ell} = \frac{0.5/100\,\text{mm}}{0.25\text{ mm}}$

error due to d measurement $2\frac{\Delta d}{d} = \frac{2 \times \frac{0.5}{100}}{0.5\text{ mm}} = \frac{0.5/100}{0.25}$

So error in y due to ℓ measurement = error in y due to d measurement

Section II : Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

11. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t = 0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with x-axis. Which of the following option(s) is(are) correct for time t > 0 ?

(A) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.

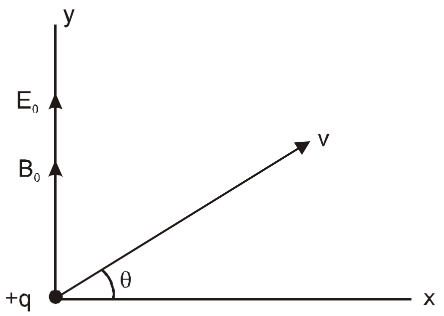
(B) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis.

(C) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(D) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

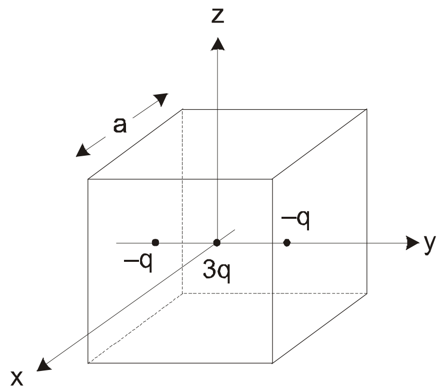
Ans. **(C), (D)**

Sol.



If $\theta = 0°$ then due to magnetic force path is circular but due to force qE_0 (↑) q will have accelerated motion along y–axis. So combined path of q will be a helical path with variable pitch so (A) and (B) are wrong.
If $\theta = 10°$ then due to $v\cos\theta$, path is circular and due to qE_0 and $v\sin\theta$, q has accelerated motion along y-axis so combined path is a helical path with variable pitch (C) is correct.
If $\theta = 90°$ then $F_B = 0$ and due to qE_0 motion is accelerated along y-axis. (D)

12. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, –q at (0, –a/4, 0), + 3q at (0, 0, 0) and –q at (0, +a/4, 0). Choose the correct option(s).



(A) The net electric flux crossing the plane x = +a/2 is equal to the net electric flux crossing the plane x = –a/2.

(B) The net electric flux crossing the plane y = +a/2 is more than the net electric flux crossing the plane y = –a/2.

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.

(D) The net electric flux crossing the plane z = +a/2 is equal to the net electric flux crossing the plane x = +a/2.

Ans. **(A), (C)**

Sol. Position of all the charges are symmetric about the planes $x = \frac{+a}{2}$ and $x = \frac{-a}{2}$. So net electric flux through them will be same.

Similarly flux through $y = \frac{+a}{2}$ is equal to flux through $y = \frac{-a}{2}$.

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{3q - q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

By symmetry flux through $z = \frac{+a}{2}$ is equal to flux through $x = \frac{+a}{2}$

13. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe.

(A) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is open.

(B) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is open.

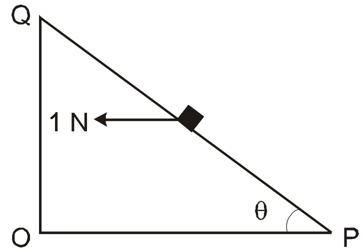
(C) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed.

(D) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed.

Ans. (B), (D)

Sol. At open end phase of pressure wave charge by π so compression returns as rarefraction. While at closed end phase of pressure wave does not change so compression return as compression.

14. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N on the block through its center of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2)



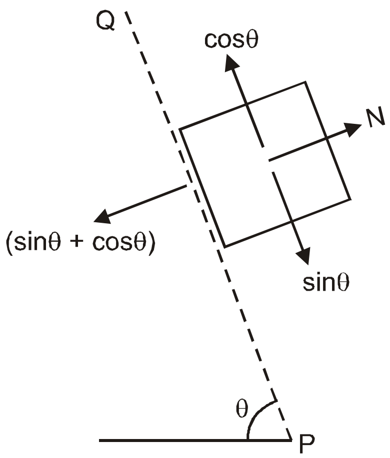
(A) $\theta = 45°$

(B) $\theta > 45°$ and a frictional force acts on the block towards P.

(C) $\theta > 45°$ and a frictional force acts on the block towards Q.

(D) $\theta < 45°$ and a frictional force acts on the block towards Q.

Ans. (A), (C)

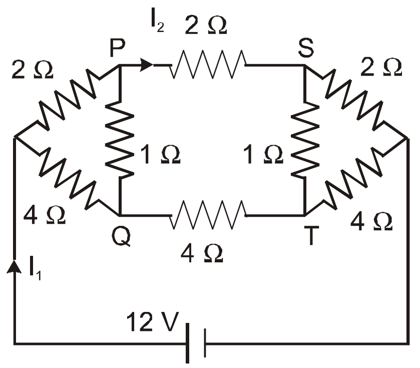


f = 0, If $\sin\theta = \cos\theta \Rightarrow \theta = 45°$

f towards Q, $\sin\theta > \cos\theta \Rightarrow \theta > 45°$

f towards P, $\sin\theta < \cos\theta \Rightarrow \theta < 45°$

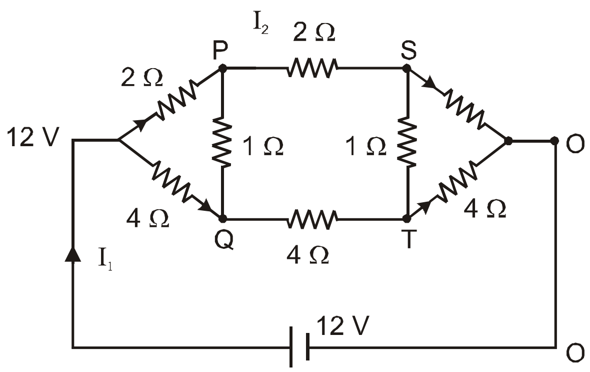
15. For the resistance network shown in the figure, choose the correct option(s).

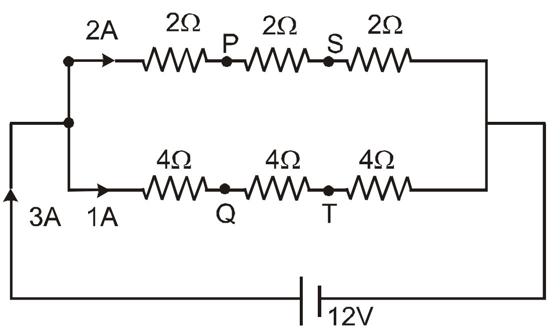


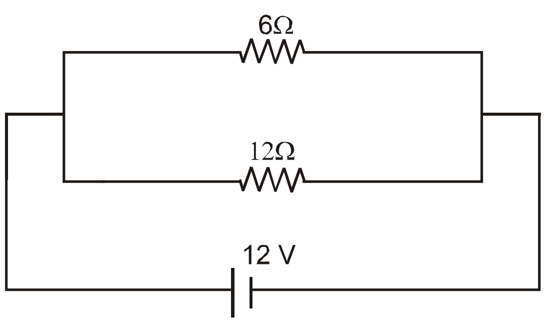
(A) The current through PQ is zero.
(B) I_1 = 3 A.
(C) The potential at S is less than that at Q.
(D) I_2 = 2 A.

Ans. **(A), (B), (C), (D)**

Sol. Due to input and output symmetry P and Q and S and T have same potential.







$R_{eq} = \frac{6 \times 12}{18} = 4\Omega$

$I_1 = \frac{12}{4} = 3A$

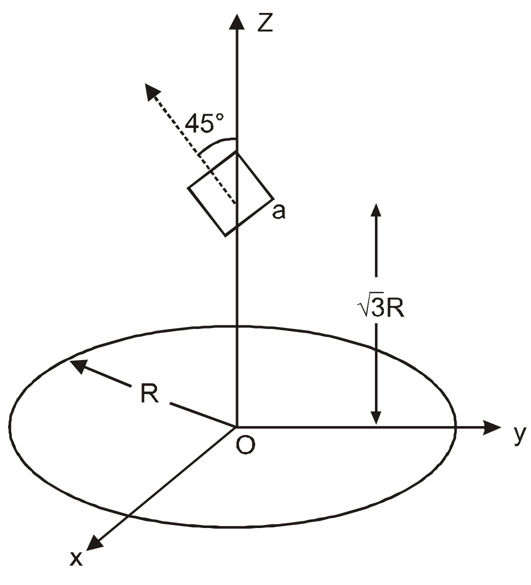
$I_2 = \left(\frac{12}{6+12}\right) \times 3$

$I_2 = 2$ A
$V_A - V_S = 2 \times 4 = 8V$
$V_A - V_T = 1 \times 8 = 8V$
$V_P = V_Q \Rightarrow$ Current through PQ $= 0$ (A)
$V_P = V_Q \Rightarrow V_Q > V_S$ (C)
$I_1 = 3A$ (B)
$I_2 = 2A$ (D)

Section III : Integer Answer Type

This section contains **5 question**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

16. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop os side a (a << R) having two turns is placed with its center at a = $\sqrt{3}$ R along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is



Ans. **P = 7**

Sol. $B = \dfrac{\mu_0 i R^2}{2(R^2 + X^2)^{3/2}}$

$$B = \frac{\mu_0 i R^2}{2(R^2 + 3R^2)^{3/2}} = \frac{\mu_0 i R^2}{2(4R^2)^{3/2}}$$

$$= \frac{\mu_0 i R^2}{2 \,.\, 2^3 \,.\, R} = \frac{\mu_0 i}{16R}$$

ϕ = NBA cos45°

$$= 2\frac{\mu_0 i}{16R} a^2 \frac{1}{\sqrt{2}}$$

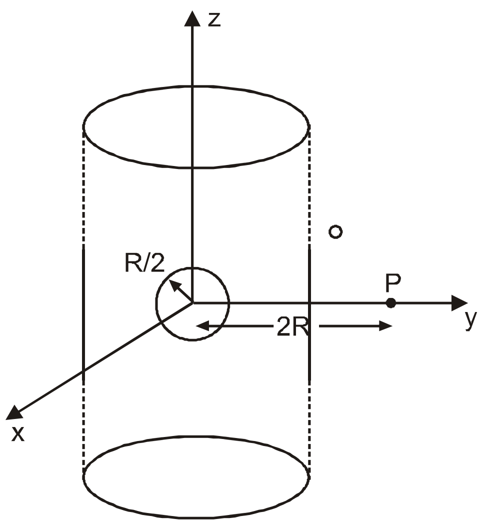
$$\phi = \frac{\mu_0 i a^2}{8\sqrt{2}R}$$

$$M = \frac{\phi}{i}$$

$$M = \frac{\mu_0 a^2}{2^{7/2}R} = \frac{\mu_0 a^2}{2^{P/2}R}$$

P = 7

17. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\dfrac{23\rho R}{16k\varepsilon_0}$. The value of k is



Ans. **K = 6**

$$E_1 = \frac{\rho . R^2}{\varepsilon_0 . 2R}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} . \frac{\rho . \frac{4}{3}\pi . \frac{R^3}{8}}{(2R)^2}$$

$$E_1 - E_2 = \frac{\rho R}{4\varepsilon_0} - \frac{\rho . R}{\varepsilon_0 . 24 \times 4}$$

$$= \frac{\rho R}{4\varepsilon_0}\left[1 - \frac{1}{24}\right]$$

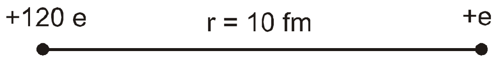
$$= \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{16K\varepsilon_0} \qquad \Rightarrow \qquad K = 6$$

18. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Brogle wavelength (in units of fm) of the proton at its start is :

(take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg, $h/e = 4.2 \times 10^{-15}$ J.s/C ; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F ; 1 fm = 10^{-15} m)

Ans. **7 fm**

Sol.



$$\frac{(9 \times 10^9)(120e)(e)}{10 \times 10^{-15}} = \frac{p^2}{2m}$$

$$\lambda = \frac{h}{p} \quad \therefore \; p^2 = \frac{h^2}{\lambda^2}$$

$$2\left(\frac{5}{3} \times 10^{-27}\right)10^{15}(9 \times 10^9)(12)e^2 = \frac{h^2}{2m\lambda^2}$$

$(120)\,(3)10^{-27+15+9}$ $\qquad \lambda^2 = (4.2)^2 \times 10^{-30}$

$$\lambda^2 = \frac{4.2 \times 4.2 \times 10^{-30}}{360 \times 10^{-3}}$$

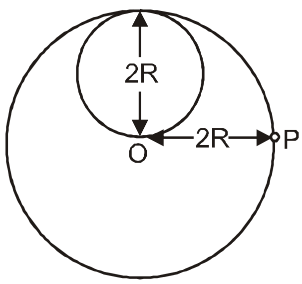
$$= \frac{42 \times 42}{360} \times 10^{-29}$$

$= 7^2 \times 10^{-30}$

$\lambda = 7 \times 10^{-15}$ m

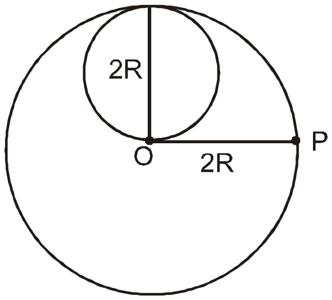
= 7 fm

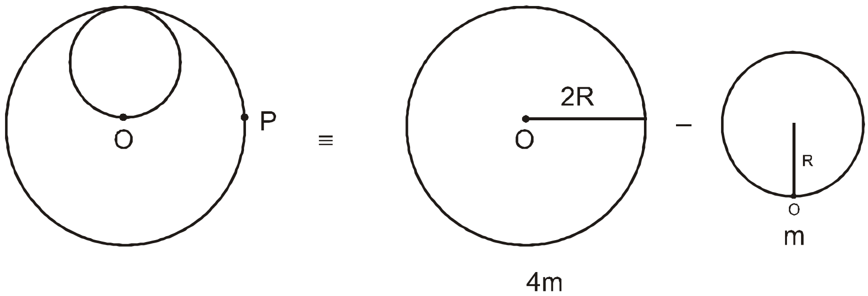
19. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is :



Ans. **3**

Sol.

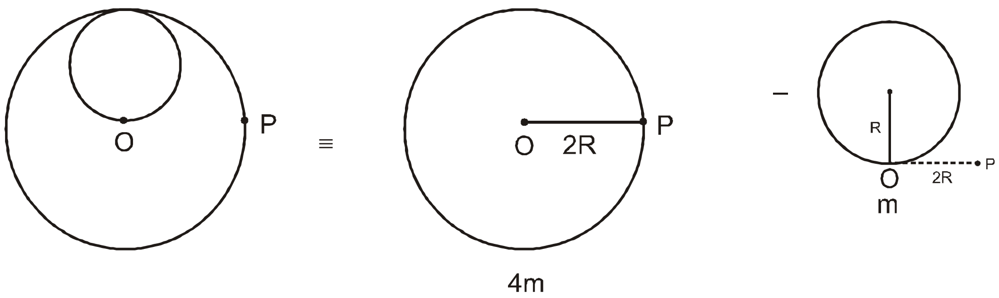




$$I_0 = \frac{(4m)\,(2R)^2}{2} - \frac{3}{2}\,mR^2$$

$$= mR^2\left[8 - \frac{3}{2}\right]$$

$$= \frac{13}{2}mR^2$$



$$I_P = \frac{3}{2}\,(4m)\,(2R)^2 - \left[\frac{mR^2}{2} + m[(2R)^2 + R^2]\right]$$

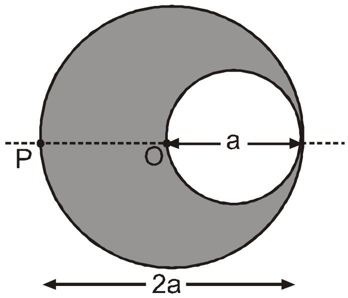
$$= 24\,mR^2 - \frac{11}{2}mR^2$$

$$= \frac{37}{2}mR^2$$

$$\frac{I_P}{I_O} = \frac{\frac{37}{2}}{\frac{13}{2}} = \frac{37}{13} \approx 3$$

Ans. 3

20. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0$ aJ, then the value of N is :



Ans. **N = 5**

Sol. $B_1 = \frac{\mu_0 J_a}{2} - \frac{\mu_0 J_a}{12}$

$= \left(\frac{\mu_0 Ja}{2}\right)\left(1 - \frac{1}{6}\right) = \frac{5}{6}\left(\frac{\mu_0 Ja}{2}\right) = \frac{5\mu_0 aJ}{12} = \frac{N}{12}\mu_0 aJ$

N = 5