# MODEL PAPER-I

Time : 3 Hours Max. Marks : 60

Instructions :

- Each unit carries 12 Marks
- Answer all units choosing one question from each unit
- All parts of the unit must be answered in one place only.
- Figures in the right hand margin indicate marks allotted.

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## UNIT – I

1. a) Find the differential equation of family of circles passing through the origin and having centre on the x- axis. 6

   b) Solve $2x\frac{dy}{dx} - y = 10x^3y^5$ 6

**OR**

2. a) Solve: $xy(1+xy^2)\frac{dy}{dx} = 1$. 6

   b) A body originally at $80^oC$ cools down to $60^oC$ in 20 minutes, the temperature of air being $40^oC$. What will be the temperature of the body after 40 minutes from the original? 6

## UNIT – II

3. a) Solve $(D^2-4D+4)y = 8x^2e^{2x}\sin 2x$. 6

   b) $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin[2\log(1+x)]$ 6

**OR**

4. a) Solve $y^{11} - 2y^1 + 2y = e^x\tan x$ by method of variation of parameters. 6

   b) Show that the frequency of free vibrations in a closed electrical circuit with inductance L and capacity C in series is $30/\pi\sqrt{LC}$ per minute. 6

## UNIT – III

5. a) Verify Lagranges mean value theorem for $f(x) = (x-1)(x-2)(x-3)$ in (0,4) 6

   b) If x is positive show that $x > \log(1+x) > x - \frac{x^2}{2}$ 6

**OR**

6. If a < b, prove that $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$ Using Lagranges mean value theorem and deducve the following :

(i) $\frac{\pi}{4}+\frac{3}{25} < \mathrm{Tan}^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$

(ii) $\frac{5\pi+4}{20} < \mathrm{Tan}^{-1}2 < \frac{\pi+2}{4}$ 6

b) Find "c" of Cauchy's mean value theorem for $f(x) = \sqrt{x}$ and

$g(x) = \frac{1}{\sqrt{x}}$ $\forall x \in [a,b]$, where $0 < a < b$. 6

**UNIT – IV**

7. State the Leibnitz test for the convergence of an alternating series.

Using this test, find the nature of convergence of

$1 - \frac{x}{1^2} + \frac{x^2}{2^2} - \frac{x^3}{3^2} + \ldots\ldots..$ 6

b) Discuss interval of convergence of the power series $\sum_{n=0}^{\infty}(-1)^n(x-1)^n$ 6

**OR**

8. a) Use Cauchy's root test to find the convergence of the

series $\sum\left(\frac{n+1}{2n+5}\right)^n x^n$ 6

b) State Raabe's test .Test the convergence of the infinite series

$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\frac{4!}{5^4}x^4+\ldots\ldots..\infty$ 6

**UNIT – V**

9. a) Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

and find its inverse 6

b) Test for consistency and solve $2x - 3y + 7z = 5$, $3x + y - 3z = 13$,

$2x + 19y - 47z = 32$. 6

**OR**

10. Reduce the quadratic form $2xy + 2yz + 2zx$ into Canonical form and find its nature. Also calculate $A^4$. 12

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**MODEL PAPER-II**

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**UNIT – I**

1 a) Solve ysin2xdx – ( $y^2$ + $\cos^2$x)dy = 0. 6

b) Solve $\frac{dy}{dx} = x^3 y^3 - xy$ 6

**OR**

2. a) Solve ($x^2$- $y^2$) dx - xy dy = 0. 6

b) A radioactive substance disintegrates at a rate proportional to its mass. When mass is 10mgm, the rate of disintegration is 0.051 mgm per day. How long it will take for the mass to be reduced from m10mgm to 5mgm. 6

**UNIT – II**

3. a) Solve $\frac{d^3y}{dx^3} - 2\frac{dy}{dx} + 4y = e^x \sin x$.. 6

b) $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$. 6

**OR**

4 a) Solve by the method of variation of parameters ($D^2$-2D)y = $e^x$ sinx. 6

b) The damped LCR circuit is governed by the equations

$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$ where L, C, R, are positive constants. Find the conditions under winch the circuit is over damped, under damped and critically damped. Find also the critical resistance. 6

**UNIT – III**

5a) Verify Rolle's theorem for $f(x) = x^2(1-x^2)$ in [0,1]. 6

b) Obtain Taylor's series expansion of f(x) = cosx in powers of x. 6

**OR**

6a) Using Cauchy's mean value theorem prove that $\dfrac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$,

where $0 < \alpha < \theta < \beta < \pi/2$.

6

b) Obtain Maclaurin's series expansion of $f(x) = (1+x)^n$. 6

**UNIT – IV**

7 a) If $\sum u_n$ converges then prove that $\lim\limits_{n\to\infty} u_n = 0$. 6

b Discuss the convergence of $\sum \sin\dfrac{1}{n}$. 6

**OR**

1 a) ) Discuss the convergence of $\sum \dfrac{\sqrt{n}}{n^2+1}$. 6

b) Discuss the convergence of $\sum \left(\dfrac{n+1}{n+2}\right)^n x^n \, (x > 0)$. 6

**UNIT – V**

2 a) Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

and find its inverse. 6

b) For what values of $\lambda$ and $\mu$ the equations x + y + z = 6, x + 2y + 3z = 10, x + 2y +$\lambda$z = $\mu$ habe (i) no solution, (ii) unique solution , (iii) an infinite number of solutions 6

**OR**

3 Reduce the quadratic form $3x^2+5y^2+3z^2-2xy-2yz+2zx$ into Canonical form and specify the matrix of transformation 12

**MODEL PAPER-III**

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**UNIT – I**

1 a) Solve the differential equation $\frac{dy}{dx}\left(x^2y^3+xy\right)=1$. 6

b) Solve the differential equation $y^2dx+\left(x^2-xy-y^2\right)dy=0$. 6

**OR**

2. a) Solve the differential equation $\frac{dy}{dx}=\frac{y+x-2}{y-x-4}$. 6

b) Show that the system of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{Y^2}{b^2+\lambda}=1$ is self orthogonal where λ is a parameter. 6

**UNIT – II**

3. a) Solve the differential equation $(D^3+2D^2+D)y=e^{2x}+x^2+x+\sin 2x$. 6

b) Solve the differential equation $\frac{d^2y}{dx^2}+y=x\cos x$ using the method of variation of parameters. 6

**OR**

4 a) Solve $(2x-1)^3\frac{d^3y}{dx^3}+(2x-1)\frac{dy}{dx}-2y=x$. 6

b) An e.m.f. E Sin pt is applied at t=0 to a circuit containing a capacitance 'C' and inductance L. The current 'i' satisfies the equation $L\frac{di}{dt}+\frac{1}{C}\int i\,dt=E\,Sin\,pt$. If $p^2=1/(LC)$ and initially the current 'i' and the charge q are zero. Show that the current at time t is Et/2L Sin pt where i = dq/dt. 6

## UNIT – III

5a) Verify Rolle's theorem for the function $\log\left[\frac{x^2 = ab}{x(a+b)}\right]$ in the interval [a,b] where a > 0, b > 0. 6

b) Show that $\frac{\sin^{-1} x}{\sqrt{1-x^2}} = x + \frac{4x^3}{3!} + .........$ 6

**OR**

6a) Calculate $\sqrt[5]{245}$ using Lagrange mean value theorem. 6

b) Expand $\log(1+\sin^2 x)$ in terms of x upto $6^{th}$ degree. 6

## UNIT – IV

7 a) Test the convergence of the infinite series

$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^{23}}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + .........$$ 6

b) Test the absolute convergence or conditional convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n\sqrt{n}}$ 6

**OR**

8 a) ) Discuss the nature of the infinite series

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + ..........\infty$$ 6

b) Find the interval of convergence of $\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + .........$ 6

## UNIT – V

9 a). Find the values of k for which the system of equations x + y + z = 1, x + 2y + 4z = k, x + 4y +10z = $k^2$ is consistent and solve for each possible value of k. 6

b) Verify Cayley- Hamilton theorem for $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and find its inverse. 6

**OR**

10 If A = $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ calculate $A^4$ by diagonalising A. 12