Model Question Paper (Theory)
B.A/B.Sc. III Year Examination, March/April 2011
MATHEMATICS PAPER-III

Time:3Hrs **Maximum Marks:100**

NOTE: Answer 6 questions from Section- A and 4 questions from Section –B choosing atleast one from each unit. Each question in Section- A carries 6 marks and each question in Section-B carries 16 marks.

SECTION-A (6×6=36)

UNIT-I

1) Define a subspace. Prove that the intersection of two subspaces is again a subspace.

2) Define Linear transformation. Show that the mapping $T: V_3(R) \to V_2(R)$ defined as $T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3,\ a_1 - 3a_2 - 2a_3)$ is a linear transformation from $V_3(R)$ in to $V_2(R)$.

UNIT-II

3) Find all eigen values of the matrix $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

4) Define orthogonal set. Show that any orthogonal set of non-zero vectors in an inner product space V is linearly independent.

UNIT-III

5) Evaluate $\iint x^2y^2\, dxdy$ over the domain $\{ (x,y): x \geq 0; y \geq 0; (x^2 + y^2) \leq 1\}$.

6) Evaluate $\iint (x^2 + y^2)\, dxdy$ over the domain bounded by $xy = 1; y = 0; y = x; x = 2$.

UNIT-IV

7) Define irrotational vector. Show that $A = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is Irrotational. Find φ such that $A = \nabla\varphi$.

8) Evaluate $\iint A.n\, ds$ where A=18zi-12j+3yk and S is that part of the plane 2x+3y+6z=12 which is located in first octant.

SECTION-B (4×16=64)

UNIT-I

9) a) Define Basis of a vector space. Prove that any two basis of a finite dimensional vector Space $V(F)$ have same number of elements.

b) If W_1, W_2 are two subspaces of a finite dimensional vector Space $V(F)$ then

$dim(W_1 + W_2) = dimW_1 + dimW_2 - \dim(W_1 \cap W_2)$.

10) a) State and prove Rank and Nullity theorem in linear transformation.

b) Show that linear operator T defined on R^3 by $T(x, y, z) = (x + z, x - z, y)$ is invertible. And hence find T^{-1}.

UNIT-II

11) a) Prove that distinct characteristic vectors of T corresponding to distinct characteristic of T are linearly independent.

b) Let T be the linear operator on R^3 which is represented in standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable.

12) a) State and prove schwarz's inequality .

b) Apply the Gram- Schmidt process to the vector $\beta_1 = (1,0,1)$; $\beta_2 = (1,0,-1)$; $\beta_3 = (0,3,4)$ to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

UNIT-III

13) a) Prove the sufficient condition for the existence of the integral.

b) Verify that $\iint_R (x^2 + y^2) dydx = \iint_R (x^2 + y^2) dxdy$ where the domain R is the triangle bounded by the lines $y = 0, y = x, x = 1$.

14)a) Prove the equivalence if a double integra with repeated integrals.

b) Evaluate the following integral: $\iint \frac{x-y}{x+y} dxdy$ over $[0,1; 0,1]$.

UNIT-IV

15) a) For any vector A, Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla. \mathrm{A}) - \nabla^2 A$.

b) If $U = 3x^2y; V = xz^2 - 2y$. Evaluate $grad\,[\,(gradU)\,.\,(gradV)\,]$.

16) a) State and prove Green's theorem in a plane.

b) Verify stoke's theorem for $A = (2x - y)i - yz^2j - y^2zk$. where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.

DEPARTMENT OF MATHEMATICS

B.A. / B.Sc.-III (Practical) Examination 2010-2011

Subject : MATHEMATICS (New Syllabus)

Paper : IV(a) Numerical Analysis

QUESTION BANK

Time : 3 hours **Marks : 50**

UNIT-I

1) Define the term percentage error. If $u = 3v^7 - 6v$ Find the percentage error in u at $v = 1$, if the error in v is 0.05.
2) Define the terms absolute and relative errors. If $y = \frac{0.31x+2.73}{x+0.35}$, where the coefficients are rounded off. Find the absolute and relative error in y when $x = 0.5 \pm 0.1$
3) If $u = \frac{5xy^2}{z^3}$ then find maximum relative error at $\Delta x = \Delta y = \Delta z = 0.001$ and $x = y = z = 1$
4) Find the real root of $x^3 - x - 1 = 0$, using Bisection method.
5) Find the real root of $x^3 - x^2 - 1 = 0$ up to three decimal places using Bisection method.
6) Use iterative method to find a real root of the following equation, correct to four decimal places $x = \frac{1}{(x+1)^2}$.
7) Use iterative method to find a real root of the following equation, correct to four decimal places $x = (5 - x)^{\frac{1}{3}}$.
8) Use iterative method to find a real root of the following equation, correct upto four decimal places $sinx = 10(x - 1)$.
9) Establish the formula $x_{i+1} = \frac{1}{2}(x_i + \frac{N}{x_i})$ and hence compute the value of $\sqrt{2}$ correct to six decimal places.

   Use newton Raphson method to obtain a root and correct to three decimal places of the following equations:

10) $sinx = 1 - x$ 11)$x^4 + x^2 - 80 = 0$ 12)$3x = cosx + 1$.

13) Find $\sqrt[3]{12}$ by Nweton's method.

14) Find a double root of $x^3 - 3x^2 + 4 = 0$ by Generalised Newton's method.

15) Using Ramanujan's method find a real root of the equation $xe^x = 1$.

16) Find the root of the equation $sinx = 1 - x$ by Ramanujan's method.

17) Find the smallest root of the equation $f(x) = x^3 - 6x^2 + 11x - 6 = 0$.

18) Using Ramanujan's method, find the real root of the equation

$$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \cdots = 0.$$

19) Find the root of the equation $f(x) = x^3 - 2x - 5 = 0$ which lies between 2 & 3 by Muller's method.

20) Use Muller's method to find a root of the equation $x^3 - x - 1 = 0$.

UNIT-I I

21) Using the difference operator prove the following (i) $\mu = \sqrt{1 + \frac{\delta^2}{4}}$

(ii) $1 + \mu^2\delta^2 = (1 + \frac{\delta^2}{2})^2$

22) Find u_6 if $u_0 = -3, u_1 = 6, u_2 = 8, u_3 = 12$ and 3rd differences are constant.

23) Find a cubic polynomial which takes the values

x	0	1	2	3	4	5
y	1	2	4	8	15	26

24) If $y_0 = 2649$, $y_2 = 2707$, $y_3 = 2967$, $y_4 = 2950$, $y_5 = 2696$ and $y_6 = 2834$ then find y_1.

25) Prove the following

a) $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \cdots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n}.$

b) $u_x + x_{c_1}\Delta^2 u_{x-1} + x_{c_2}\Delta^4 u_{x-2} + \cdots = u_0 + x_{c_1}\Delta u_1 + x_{c_2}\Delta^2 u_2 + \cdots$

26) From the following table ,find the number of students who secured mark between 60 and 70.

Marks obtained	0-40	40-60	60-80	80-100	100-120
Number of students	250	120	100	70	50

27) Find the cubic polynomial which takes the values :

$y(1) = 24,\ \ y(3) = 120,\ \ y(5) = 336,\ \ y(7) = 720.$ Hence obtain $y(8)$.

28) The following data gives the melting point of an alloy of lead and zinc;θ is the temperature in degree centigrade;x is the percent of lead.Find θ when$x = 84$.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

29) From the following table, find the value of$e^{1.17}$ by using Gauss forward formula.

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

30) The following values of x and y are given .Find the value of $y(0.543)$.

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$y(x)$	2.631	3.328	4.097	4.944	5.875	6.896	8.013

31) Use Gauss interpolation formula to find y_{41} with help of following data

$y_{30} = 3678.2,\ \ y_{35} = 2995.1,\ \ y_{40} = 2400.1,\ \ y_{45} = 1876.2,\ \ y_{50} = 1416.2$

32) By using central difference formula find the value of log 337.5 satisfying the following table

x	310	320	330	340	350	360
$logx$	2.4014	2.5052	2.5185	2.5315	2.5441	2.5563

33) Values of $y = \sqrt{x}$ are listed in the following table, which are rounded off to 5 decimal places. Find $\sqrt{1.12}$ by using Stirling's formula

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
$y = \sqrt{x}$	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

34) By using Lagrange's formula, express the following rational fraction as sum of partial fractions $\frac{x^2+6x+1}{(x^2-1)(x^2-10x+24)}$.

35) By means of Lagrange's formula prove that approximately

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$$

36) Apply Lagrange's formula to find the root of $f(x) = 0$ when

$f(30) = -30,\ f(34) = -13,\ f(38) = 3, f(42) = 18.$

37) Use Stirling's formula to find u_{32} for the following table

$u_{20} = 14.035,\ u_{25} = 13.674,\ u_{30} = 13.257,\ u_{35} = 12.734,\ u_{40} = 12.089,$
$u_{45} = 11.309.$

38) Construct the divided difference table for the given data and evaluate $f(1)$.

x	-4	-2	-1	0	2	5	10
$f(x)$	469	47	7	1	-5	271	7091

39) Use Newton's divided difference interpolation to obtain a polynomial $f(x)$ satisfying the following data of values and hence find$f(5.)$

x	-1	0	3	6	7
$f(x)$	3	-6	39	822	1611

40) Prove that the third order divided difference of the function $f(x) = \frac{1}{x}$ with arguments a, b, c, d is $-\frac{1}{abcd}$.

UNIT-III

41) Fit a straight line of the form $y = a + bx$ to the data

x	0	5	10	15	20	25	30
y	10	14	19	25	31	36	39

42) Find best values of a, b, c so that the parabola $y = a + bx + cx^2$ fits the data

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.2	1.5	2.6	2.8	3.3	4.1

43) Fit a second degree parabola of the $y = ax^2 + bx + c$ to the following data.

x	0	1	2	3	4
y	1	5	10	22	38