50 Keys To CAT

Arithmetic, Algebra, Geometry and Modern Mathematics

A collection of 50 very important formulae for Quantitative Ability Section of CAT from the pioneers of online CAT training

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1. Averages

$$\text{Simple Average} = \frac{\text{Sum of Elements}}{\text{Number of Elements}}$$

$$\text{Weighted Average} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_nx_n}{w_1 + w_2 + w_3 + \cdots + w_n}$$

2. Mean

$$\text{Arithmetic Mean} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

$$\text{Geometric Mean} = \sqrt[n]{x_1 \times x_2 \times x_3 \times \ldots \times x_n}$$

$$\text{Harmonic Mean} = \frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}\right)}$$

For two numbers a and b,

$$\text{Harmonic Mean} = \frac{2ab}{a + b}$$

3. Percentage Change

$$\text{Percentage Change} = \frac{\text{Final Value} - \text{Initial Value}}{\text{Initial Value}} \times 100$$

For two successive changes of $a\%$ and $b\%$,

$$\text{Total Percentage Change} = \left(a + b + \frac{ab}{100}\right)\%$$

4. Interest

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} = \frac{P \times R \times N}{100}$$

$$\text{Compound Interest} = P \times \left(1 + \frac{R}{100}\right)^N - P$$

Amount = Principal + Interest

5. Population Formula

$$P' = P \times \left(1 \pm \frac{r}{100}\right)^n$$

[Here, P = Original population, P' = population after n years, r% = rate of annual change]

6. Depreciation Formula

$$P' = P \times \left(1 - \frac{r}{100}\right)^n$$

[Here, P = original value, P' = final value after n years, r% = rate of annual depreciation]

7. Growth

Absolute Growth = Final Value – Initial Value

$$\text{Growth rate for a year} = \frac{\text{Final Value} - \text{Initial Value}}{\text{Initial Value}} \times 100$$

$$\text{S. A. G. R. or A. A. G. R.} = \frac{\text{Growth Rate}}{\text{Number of Years}} \times 100$$

$$\text{C. A. G. R.} = \left(\frac{\text{Final Value}}{\text{Initial Value}}\right)^{\frac{1}{\text{Number of Years}}} - 1$$

[Here, S. A. G. R. = Simple Annual Growth Rate, A. A. G. R. = Average Annual Growth Rate and C. A. G. R. = Compound Annual Growth Rate]

8. Profit and Loss

Profit = SP − CP

Loss = CP − SP

$$\text{Percentage Profit} = \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100$$

$$\text{Percentage Loss} = \frac{\text{Loss}}{\text{CP}} \times 100 = \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100$$

9. False Weights

If an item is claimed to be sold at cost price, using false weights, then the overall percentage profit is given by

$$\text{Percentage Profit} = \left(\frac{\text{Claimed weight of item}}{\text{Actual weight of item}} - 1\right) \times 100$$

10. Discount

Discount = Marked Price – Selling Price

$$\text{Discount Percentage} = \frac{\text{Discount}}{\text{Marked Price}} \times 100$$

Buy x and Get y Free

If articles worth Rs. x are bought and articles worth Rs. y are obtained free along with x articles, then the discount is equal to y and discount percentage is given by

$$\text{Percentage discount} = \frac{y}{x+y} \times 100$$

Successive Discounts

When a discount of a% is followed by another discount of b%, then

$$\text{Total discount} = \left(a + b - \frac{ab}{100}\right)\%$$

11. Ratios

If $a : b = c : d$, then $a : b = c : d = (a + c) : (b + d)$

If $a < b$, then for a positive quantity x,

$$\frac{a+x}{b+x} > \frac{a}{b} \text{ and } \frac{a-x}{b-x} < \frac{a}{b}$$

If $a > b$, then for a positive quantity x,

$$\frac{a+x}{b+x} < \frac{a}{b} \text{ and } \frac{a-x}{b-x} > \frac{a}{b}$$

12. Proportions

If $a : b :: c : d$ or $\frac{a}{b} = \frac{c}{d}$, then

$\frac{a}{c} = \frac{b}{d}$ … Alternendo Law

$\frac{b}{a} = \frac{d}{c}$ … Invertendo Law

$\frac{a+b}{b} = \frac{c+d}{d}$ … Componendo Law

$\frac{a-b}{b} = \frac{c-d}{d}$ … Dividendo Law

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$ … Componendo and Dividendo Law

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = k$, then $\frac{a+c+e+\cdots}{b+d+f+\cdots} = k$

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = k$ and p, q, r are real numbers, then $\frac{pa^n + qc^n + re^n + \cdots}{pb^n + qd^n + rf^n + \cdots} = k^n$

13. Successive Replacement

$$\frac{\text{Quantity of milk remaining after } n^{\text{th}} \text{ replacement}}{\text{Quantity of total mixture}} = \left(\frac{x-y}{x}\right)^n$$

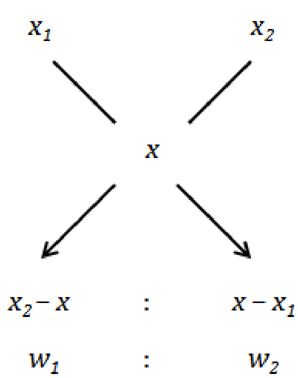
Where x is the original quantity, y is the quantity that is replaced and n is the number of times the replacement process is carried out.

14. Alligation Rule

The ratio of the weights of the two items mixed will be inversely proportional to the deviation of attributes of these two items from the average attribute of the resultant mixture.

$$\frac{w_1}{w_2} = \frac{(x_2 - x)}{(x - x_1)}$$

Alligation Cross:



15. Time, Speed and Distance

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Important Conversion Factors:

$$1 \text{ km/hr} = \frac{5}{18} \text{m/s} \;\text{ and }\; 1 \text{ m/s} = \frac{18}{5} \text{km/hr}$$

16. Average Speed

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{d_1 + d_2 + d_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

If the distance is constant, then average speed is given by harmonic mean of two speeds:

$$S_{avg} = \frac{2S_1S_2}{S_1 + S_2}$$

If the time is constant, then average speed is given by arithmetic mean of two speeds:

$$S_{avg} = \frac{S_1 + S_2}{2}$$

17. Relative Speed

For Trains

$$\text{Time} = \frac{\text{Sum of the lengths}}{\text{Relative speed}} = \frac{L_1 + L_2}{S_1 \pm S_2}$$

For Boats and Streams

$$S_{downstream} = S_{boat} + S_{stream}$$

$$S_{upstream} = S_{boat} - S_{stream}$$

18. Time and Work

$$\text{Number of days to complete the work} = \frac{1}{\text{Work done in one day}}$$

19. Application of H.C.F.

The greatest natural number that will divide x, y and z leaving remainders r_1, r_2 and r_3, respectively, is the H.C.F. of $(x - r_1)$, $(y - r_2)$ and $(z - r_3)$

20. Application of L.C.M.

The smallest natural number that is divisible by x, y and z leaving the same remainder r in each case is the L.C.M. of (x, y and z) + r

21. H.C.F. and L.C.M. of Fractions

$$\text{H. C. F. of fractions} = \frac{\text{H. C. F. of numerators of all fractions}}{\text{L. C. M. of denominators of all fractions}}$$

$$\text{L. C. M. of fractions} = \frac{\text{L. C. M. of numerators of all fractions}}{\text{H. C. F. of denominators of all fractions}}$$

22. Properties of Surds

$\left[\sqrt[n]{a}\right]^n = a$

$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$

$\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

23. Laws of Indices

If a and b are non-zero rational numbers and m and n are rational numbers, then

$a^0 = 1$

$a^{-m} = \dfrac{1}{a^m}$

$\sqrt[m]{a} = a^{\left(\frac{1}{m}\right)}$

$a^{m/n} = \sqrt[n]{a^m}$

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$(ab)^m = a^m b^m$

$a^{m^n} = a^{(m^n)} = a$ raised to the power (m raised to the power n)

If $a^m = a^n$, then $m = n$

If $a^m = b^m$ and $m \neq 0$, then $a = b$ if m is odd and $a = \pm b$ if m is even

24. Laws of Logarithms

$\log_b 1 = 0$

$\log_a a = 1$

$\log_a b \times \log_b a = 1$

$\log_b(m \times n) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b m^n = n \log_b m$

$\log_b m = \frac{\log_a m}{\log_a b} = \log_a m \times \log_b a$

$b^{\log_b n} = n$

If $\log_a m = \log_b n$ and if $m = n$, then a will be equal to b

If $\log_a m = \log_b n$ and if $a = b$, then m will be equal to n

25. Binomial Theorem

If n is a natural number that is greater than or equal to 2, then according to the binomial theorem:

$$(x + a)^n = {}^n\mathrm{C}_0\, x^n a^0 + {}^n\mathrm{C}_1\, x^{n-1} a^1 + {}^n\mathrm{C}_2\, x^{n-2} a^2 + {}^n\mathrm{C}_3\, x^{n-3} a^3 + \ \dots + {}^n\mathrm{C}_n\, x^0 a^n$$

Here, ${}^nC_r = \frac{n!}{(n-r)!\,r!}$

26. Roots of Quadratic Equation

The two roots of the quadratic equation, $ax^2 + bx + c = 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

27. Algebraic Formulae

$(a + b)(a - b) = a^2 - b^2$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

28. Arithmetic Progression

$T_n = a + (n - 1)d$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

29. Geometric Progression

$T_n = ar^{n-1}$

$$S_n = \frac{a(r^n - 1)}{(r - 1)}$$

$$S_\infty = \frac{a}{1 - r}, \text{for } r < 1$$

30. Harmonic Progression

$$T_n = \frac{1}{a + (n - 1)d}$$

31. Sum of Important Series

Sum of first *n* natural numbers

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

Sum of the squares of the first *n* natural numbers

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of the cubes of the first *n* natural numbers

$$1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

32. Factorial

$n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n$

$n! = n \times (n-1)!$

33. Permutations

$$^{n}\mathrm{P}_r = \frac{n!}{(n-r)!}$$

34. Combinations

$$^{n}\mathrm{C}_r = \frac{n!}{(n-r)!\,r!}$$

Important Properties:

$^{n}\mathrm{C}_r = {}^{n}\mathrm{C}_{n-r}$

$^{n}\mathrm{C}_0 + {}^{n}\mathrm{C}_1 + {}^{n}\mathrm{C}_2 + {}^{n}\mathrm{C}_3 + \ldots + {}^{n}\mathrm{C}_n = 2^n$

35. Partition Rule

Number of ways of distributing *n* identical things among *r* persons when each person may get any number of things = ${}^{n+r-1}C_{r-1}$

36. Probability

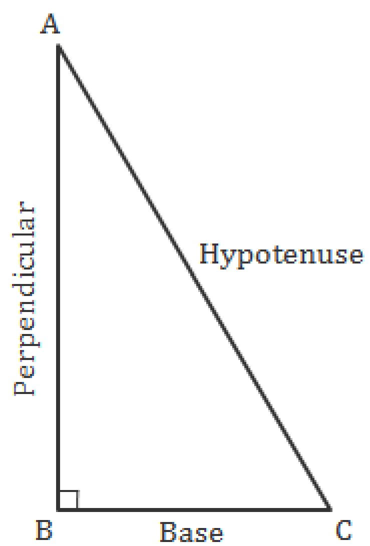
$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$$

$$\text{Odds in favour} = \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}$$

$$\text{Odds against} = \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcomes}}$$

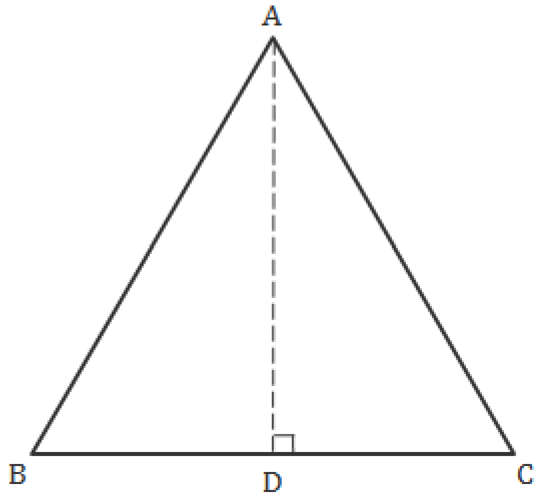
37. Pythagoras Theorem

For right triangle ABC



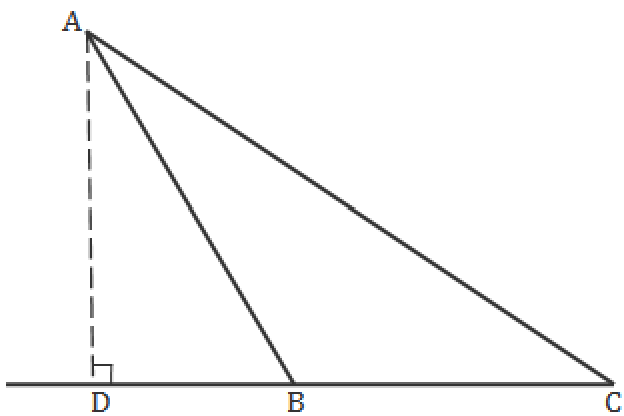
$AC^2 = AB^2 + BC^2$

For acute triangle ABC



$AC^2 = AB^2 + BC^2 - 2 \times BC \times BD$

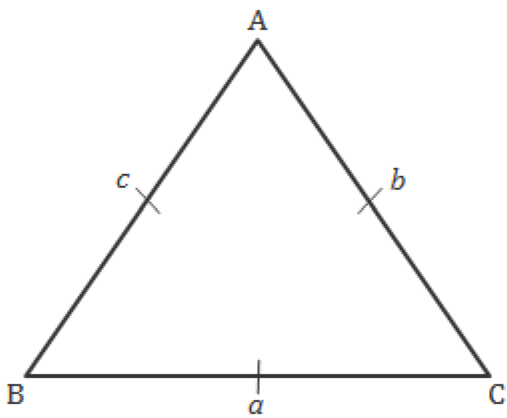
For obtuse triangle ABC



$AC^2 = AB^2 + BC^2 + 2 \times BC \times BD$

38. Area of Triangle

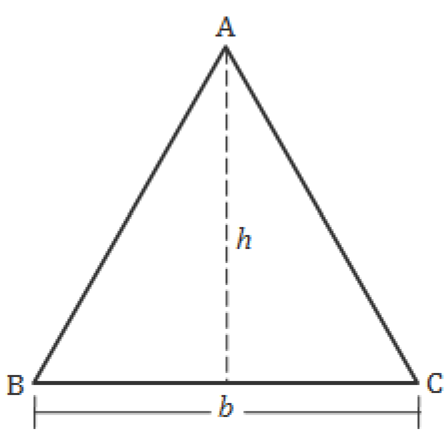
When lengths of the sides are given



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

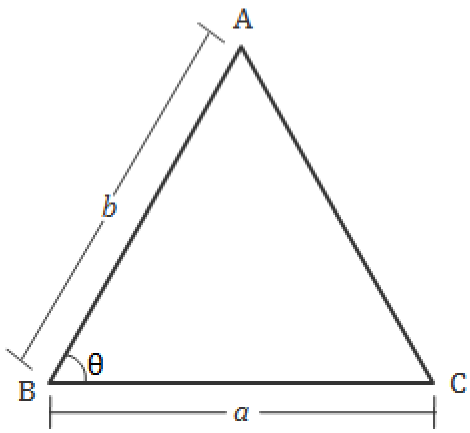
$$\text{where, semiperimeter } (s) = \frac{a+b+c}{2}$$

When lengths of the base and altitude are given



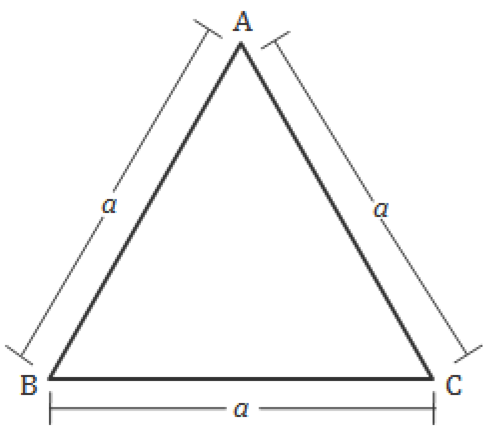
$$\text{Area} = \frac{1}{2}bh$$

When lengths of two sides and the included angle are given



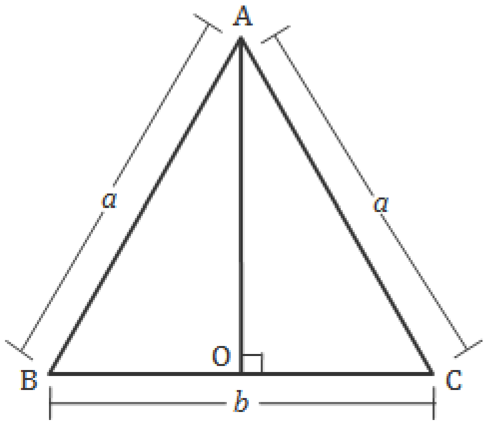
$$\text{Area} = \frac{1}{2}ab\sin\theta$$

For Equilateral Triangle



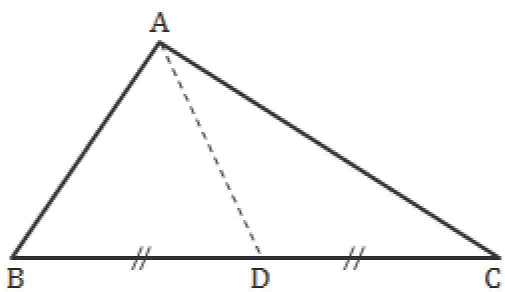
$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

For Isosceles Triangle



Area $= \frac{b}{4} \times \sqrt{4a^2 - b^2}$

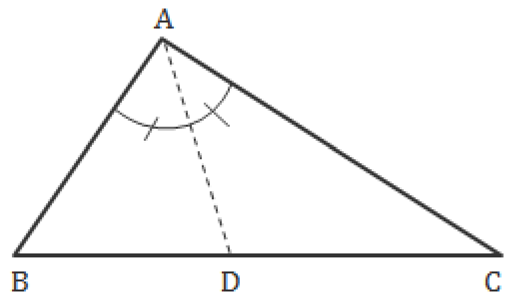
39. Apollonius Theorem



If AD is the median, then:

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

40. Angle Bisector Theorem



If AD is the angle bisector for angle A, then:

$$\frac{AB}{BD} = \frac{AC}{CD}$$

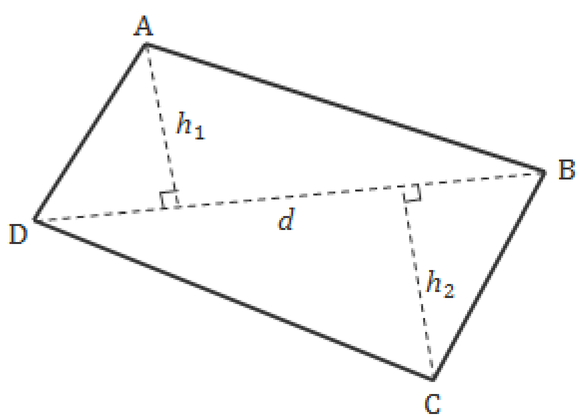
41. Area of Quadrilateral

For Cyclic Quadrilateral

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

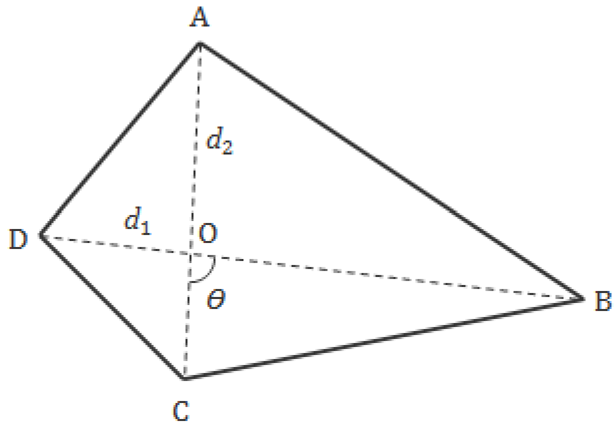
$$\text{where, semiperimeter } (s) = \frac{a+b+c+d}{2}$$

If lengths of one diagonal and two offsets are given



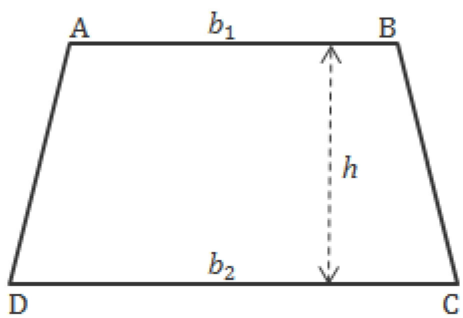
$$\text{Area} = \frac{1}{2} d(h_1 + h_2)$$

If lengths of two diagonals and the included angle are given



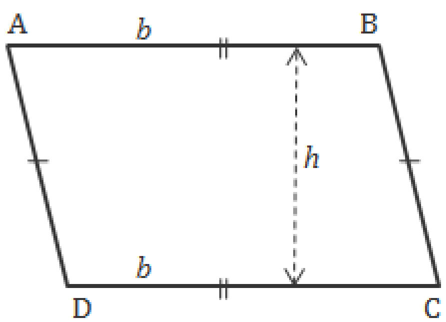
$$\text{Area} = \frac{1}{2} d_1 d_2 \sin\theta$$

For Trapezium



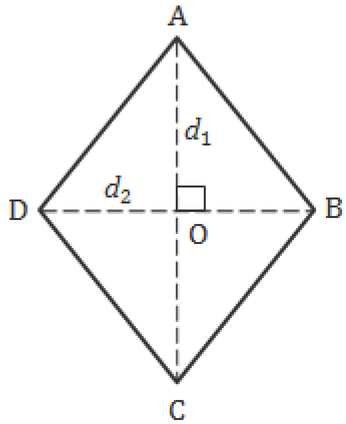
$$\text{Area} = \frac{1}{2}(b_1 + b_2)h$$

For Parallelogram



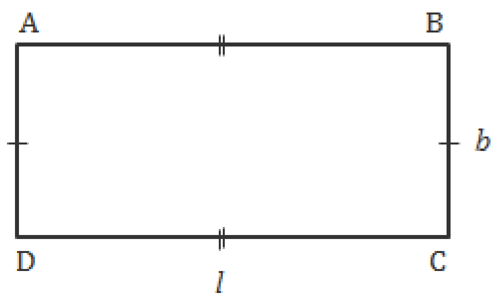
$$\text{Area} = bh$$

For Rhombus



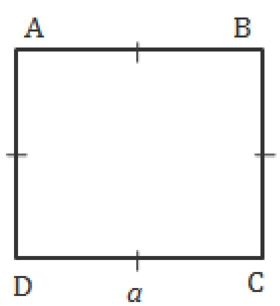
Area $= \frac{1}{2} d_1 d_2$

For Rectangle



Area $= lb$

For Square



Area $= a^2$

42. Polygons

Number of Diagonals

$$N_d = \frac{n(n-3)}{2}$$

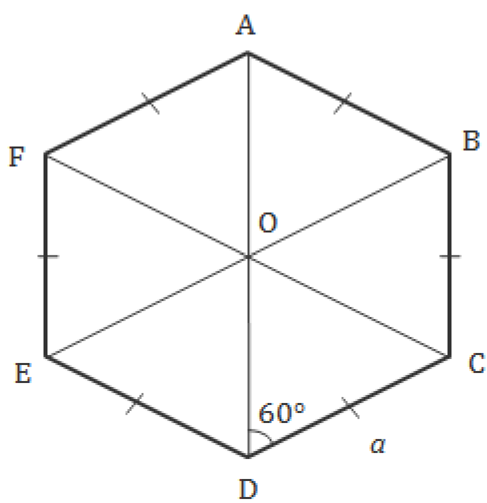
The sum of all the interior angles

$$\sum A_i = (n-2)180^\circ$$

The sum of all the exterior angles

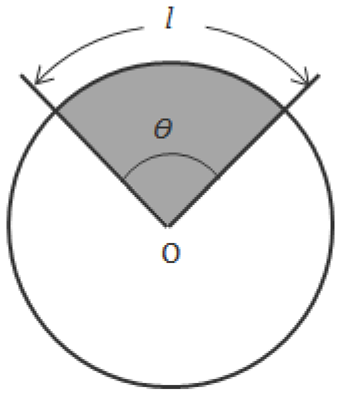
$$\sum A_e = 360^\circ$$

43. Area of Regular Hexagon



$$\text{Area} = \frac{3\sqrt{3}}{2}a^2$$

44. Circle



Circumference

$C = 2\pi r$

Area

$A = \pi r^2$

Length of Arc

$$l = 2\pi r\left(\frac{\theta}{360°}\right)$$

Area of Sector

$$A_S = \pi r^2\left(\frac{\theta}{360°}\right)$$

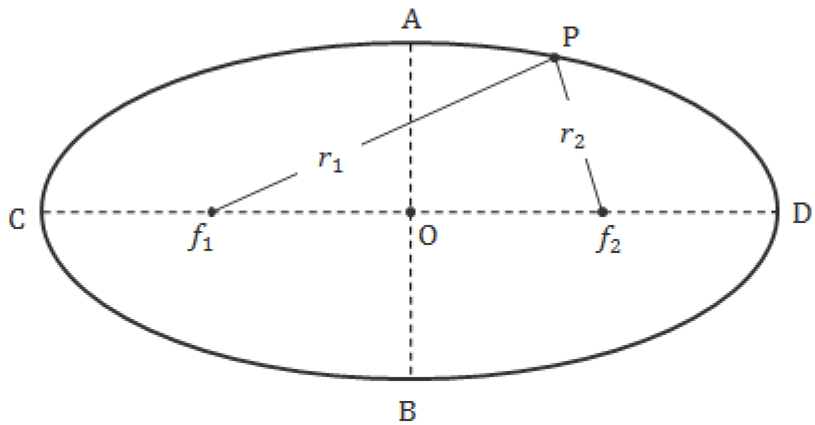
Or,

$$A_S = \frac{1}{2}lr$$

Perimeter of Sector

$P_S = l + 2r$

45. Ellipse



If semi-major axis (OD) = a and semi-minor axis (OA) = b,

Perimeter of the ellipse

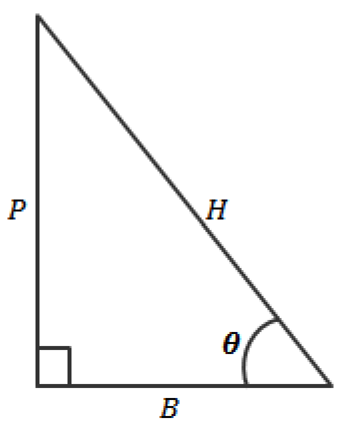
$P_e = \pi(a + b)$

Area of the ellipse

$A_e = \pi ab$

46. Trigonometric Ratios

For a right triangle, if P is the length of perpendicular, B is the length of base, H is the length of hypotenuse and $\boldsymbol{\theta}$ is the angle between base and hypotenuse,



$$\sin\theta = \frac{P}{H}$$

$$\cos\theta = \frac{B}{H}$$

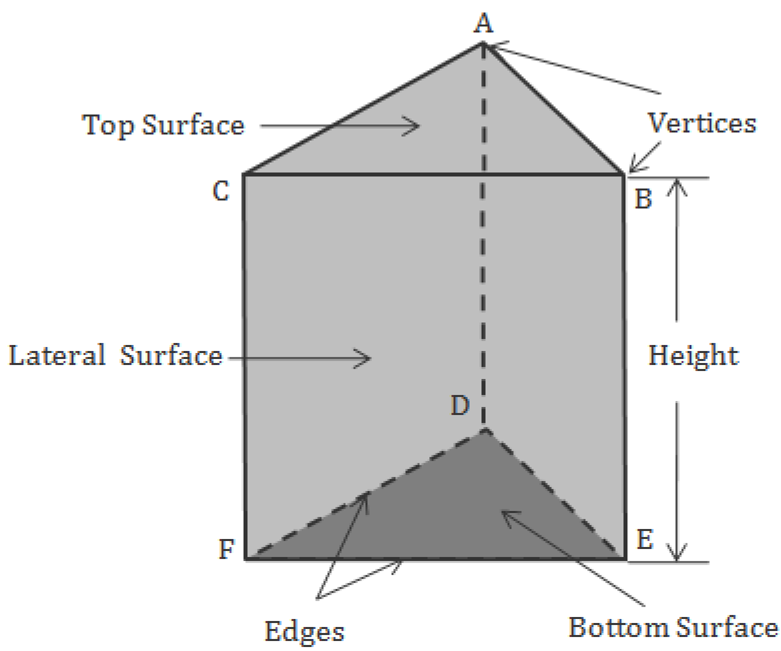
$$\tan\theta = \frac{P}{B}$$

47. Distance between Points

Distance between two points A (x_1, y_1) and B (x_2, y_2) is given by

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

48. Right Prism

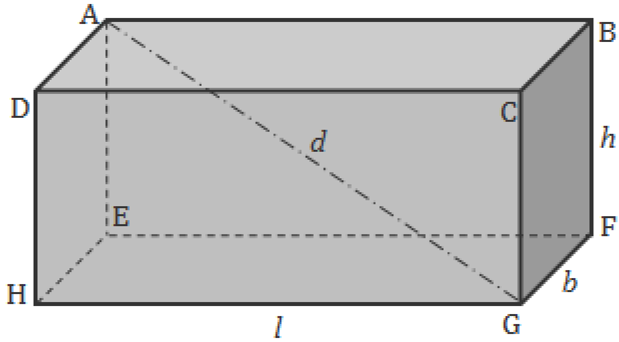


Lateral Surface Area (L.S.A.) = Perimeter of base × height

Total Surface Area (T.S.A.) = L.S.A. + 2 × Area of base

Volume (V) = Area of base × height

Cuboid



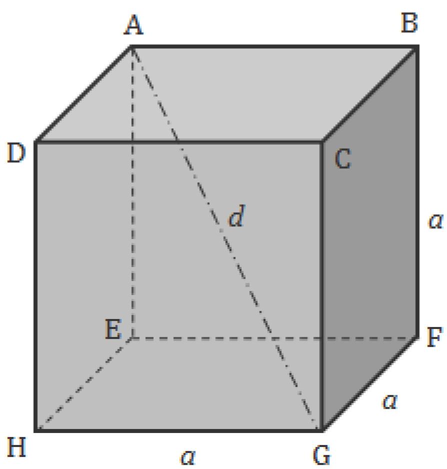
L.S.A. = $2(lh + bh)$

T.S.A. = $2(lh + bh + lb)$

Volume (V) = lbh

Body diagonal (d) $= \sqrt{l^2 + b^2 + h^2}$

Cube



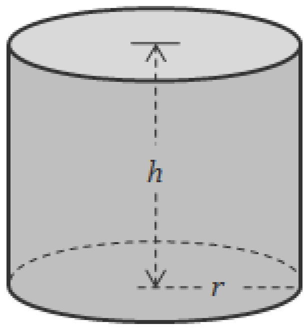
L.S.A. = $4a^2$

T.S.A. = $6a^2$

Volume (V) = a^3

Body diagonal (d) $= a\sqrt{3}$

Cylinder

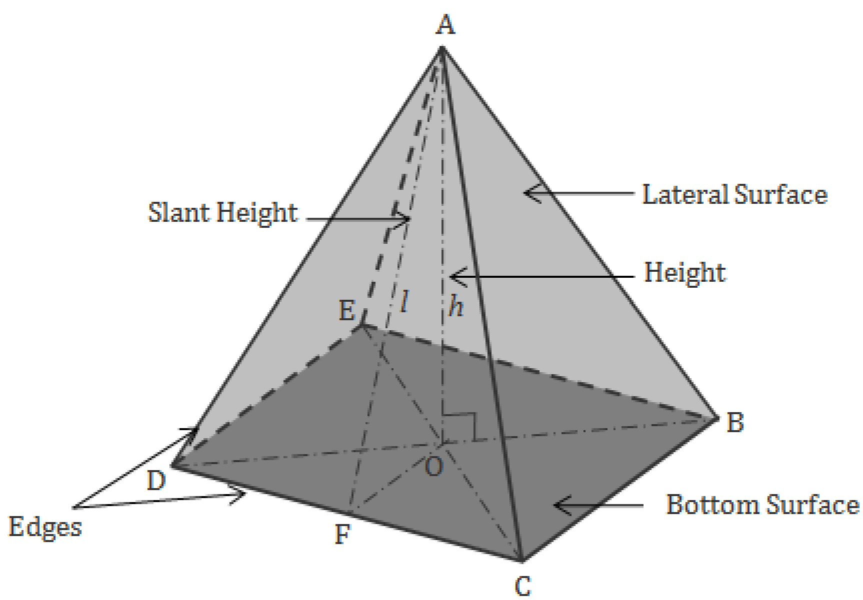


Curved Surface Area (C.S.A.) = $2\pi rh$

T.S.A. = $2\pi rh + 2\pi r^2$

Volume (V) = $\pi r^2 h$

49. Right Pyramid

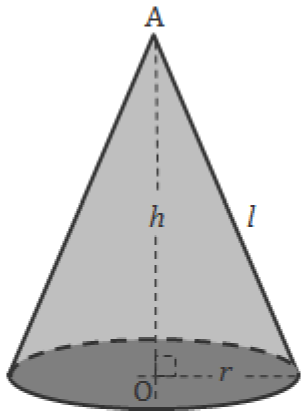


$$\text{L.S.A.} = \frac{1}{2} \times \text{Perimeter of Base} \times \text{Slant Height}$$

T.S.A. = L.S.A. + Area of base

$$\text{Volume } (V) = \frac{1}{3} \times \text{Area of Base} \times \text{Height}$$

Cone



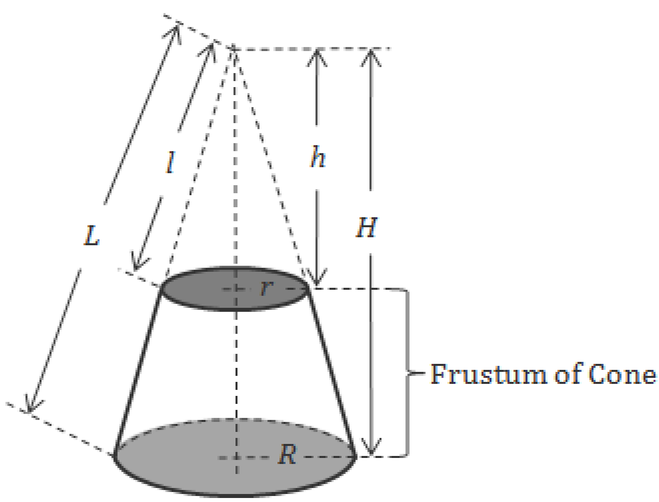
C.S.A. = $\pi r l$

T.S.A. = $\pi r l + \pi r^2$

Volume $(V) = \frac{1}{3}\pi r^2 h$

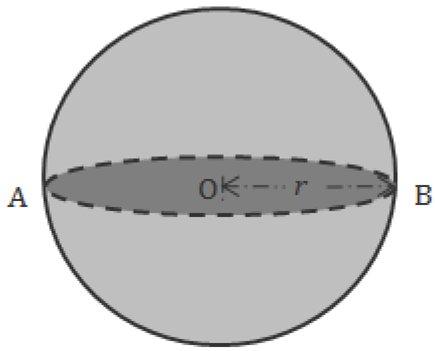
Slant height $(l) = \sqrt{(r^2 + h^2)}$

Frustum of a cone



$$\frac{\text{Volume of the original Cone}}{\text{Volume of the removed Cone}} = \frac{V}{v} = \left(\frac{R}{r}\right)^3 = \left(\frac{H}{h}\right)^3 = \left(\frac{L}{l}\right)^3$$

50. Sphere

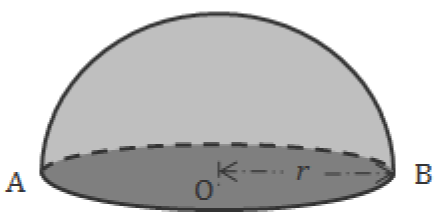


C.S.A. = $4\pi r^2$

T.S.A. = $4\pi r^2$

Volume $(V) = \frac{4}{3}\pi r^3$

Hemisphere



C.S.A. = $2\pi r^2$

T.S.A. = $3\pi r^2$

Volume $(V) = \frac{2}{3}\pi r^3$

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