Buckle Down™

Illinois ISAT

2nd EDITION

6

Mathematics

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To the Teacher:

Indicator numbers are listed for each lesson in the table of contents. The numbers in the shaded gray bar that runs across the tops of the pages in the workbook indicate the indicators for a given page (see example to the left).

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ISAT Objective: 6.6.19

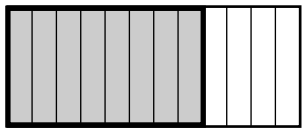
Proportions

A **proportion** states that two ratios are equal.

Example

The drawing on the left shows that $\frac{8}{12}$ of the figure is shaded.

The drawing on the right shows that $\frac{2}{3}$ of the figure is shaded.



You can see from the drawings that the ratios $\frac{8}{12}$ and $\frac{2}{3}$ are equivalent. Here is the way you show the proportion: $\frac{8}{12} = \frac{2}{3}$.

The **cross products** of a proportion are equal. Multiply the numbers that are diagonally opposite each other to get the cross products.

Example

Use cross products to show that $\frac{8}{12}$ and $\frac{2}{3}$ are equal.

$\frac{8}{12} \quad \frac{2}{3}$

$8 \cdot 3 = 12 \cdot 2$

$24 = 24$

Example

The ratios $\frac{4}{7}$ and $\frac{52}{91}$ are equivalent. Use cross products to show that the proportion is true.

$\frac{4}{7} = \frac{52}{91}$

$4 \cdot 91 = 7 \cdot 52$

$364 = 364$

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ISAT Objective: 6.6.19

A proportion can be used to find the missing value in a problem-solving situation. A **variable** is a letter that can be used to represent the missing value.

Example

Pencils are on sale, 3 for 36¢. How much money does Elysa need to buy 6 pencils?

Step 1: **Set up a proportion.**

$$\begin{matrix}\textbf{pencils} \rightarrow \\ \textbf{money} \rightarrow\end{matrix} \frac{3}{36} = \frac{6}{x} \begin{matrix}\leftarrow \textbf{pencils} \\ \leftarrow \textbf{money}\end{matrix}$$

Step 2: **Multiply to get cross products.**

$$3 \bullet x = 36 \bullet 6$$

$$3 \bullet x = 216$$

Step 3: **Solve the equation.** Divide both sides of the equation by 3 so that x is by itself on one side of the equation.

$$\frac{3 \bullet x}{3} = \frac{216}{3}$$

$$x = 72$$

Step 4: **Check to see that the cross products are equal by substituting your answer for the unknown number.**

$$\frac{3}{36} = \frac{6}{72}$$

$$3 \bullet 72 = 36 \bullet 6$$

$$216 = 216$$

Elysa needs 72¢ to buy 6 pencils.

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ISAT Objective: 6.6.19

Finding and Comparing Unit Prices

You can use proportions to solve problems with unit pricing. A **unit price** tells you the cost per gallon, pound, quart, or other unit.

A unit price may be given as, for example, $3.75 per gallon, $0.55 per pound, or $2.00 per quart.

Notice that these unit prices could be given as $3.75 per 1 gallon, $0.55 per 1 pound, or $2.00 per 1 quart.

Example

Mr. Jewett bought 12 gallons of gas for $36. What is the unit price for the amount of gas that Mr. Jewett bought?

Use the ratio of dollars to gallons of gas to find the unit price.

$$\frac{\text{dollars}}{\text{gallons of gas}} = \frac{36}{12}$$

$$\frac{36}{12} = 3$$

Mr. Jewett paid $3.00 per 1 gallon of gas.

Example

Four pounds of sugar cost $3.20. How much do 5 pounds of sugar cost?

Step 1: **Use the ratio of dollars to pounds of sugar to find the unit price (the price of 1 pound of sugar).**

$$\frac{\text{dollars}}{\text{pounds of sugar}} = \frac{3.20}{4}$$

$$\frac{3.20}{4} = 0.80 \leftarrow$$ **Use a related multiplication problem to solve. Think: $4 \cdot 0.80 = 3.20$**

Sugar costs $0.80 per pound. The unit price is $0.80.

Step 2: **Use the unit price to find the cost of 5 pounds of sugar.**

$0.80 \cdot 5 = 4.00$

So, 5 pounds of sugar cost $4.00.

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ISAT Objective: 6.6.19

Practice

Directions: For questions 1–6, determine whether the proportion is true or false. Write your answer on the line.

1. $\frac{5}{16} = \frac{12}{39}$ _____

2. $\frac{11}{32} = \frac{20}{58}$ _____

3. $\frac{2}{9} = \frac{48}{216}$ _____

4. $\frac{7}{12} = \frac{35}{60}$ _____

5. $\frac{1}{25} = \frac{6}{150}$ _____

6. $\frac{18}{41} = \frac{4}{9}$ _____

Directions: For questions 7–10, solve each proportion.

7. $\frac{3}{6} = \frac{x}{8}$ _____

8. $\frac{5}{9} = \frac{10}{x}$ _____

9. $\frac{x}{12} = \frac{15}{36}$ _____

10. $\frac{21}{x} = \frac{7}{8}$ _____

11. Nine apples cost $1.80. What is the unit price per apple? _____

12. Five gallons of gas cost $15. How much do 9 gallons of gas cost? _____

13. Ten pounds of cat food cost $15. How much do 5 pounds of cat food cost?

_____

14. Which proportion is **true**?

    A. $\frac{3}{4} = \frac{5}{6}$

    B. $\frac{2}{5} = \frac{9}{23}$

    C. $\frac{5}{8} = \frac{35}{56}$

    D. $\frac{9}{10} = \frac{99}{100}$

15. Which proportion is **false**?

    A. $\frac{10}{12} = \frac{15}{18}$

    B. $\frac{15}{20} = \frac{23}{30}$

    C. $\frac{7}{15} = \frac{21}{45}$

    D. $\frac{9}{25} = \frac{36}{100}$

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