$$ar\left(r+r^2\right)=60 \qquad \ldots\ldots \text{(i)}$$

and $a_1 \cdot a_2 \cdot a_3 = 1000$

$$a \cdot ar \cdot ar^2 = 1000$$

$$(ar)^3 = (10)^3 \Rightarrow ar = 10$$

From (i), $10\left(r+r^2\right)=60$

$$r^2+r-6=0$$

$$(r+3)(r-2)=0$$

$$r=-3,\ r=2$$

When $r=-3;\ a=-10/3<0$ (rejected) $\because \ a>0$

When $r=2;\ a=5$ $\therefore \ a_7 = ar^6 = 5(2)^6 = 320$

44.(C) $a_4 - a_7 + a_{10} = m \Rightarrow a+3d-(a+6d)+a+9d = m \Rightarrow a+64 = m$

$$S_{13} = \frac{13}{2}[2a+12d] = 13(a+6d) = 13m$$

45.(A) $1^2+2^2+3^2+\ldots..+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

$\left(\dfrac{8}{5}\right)^2+\left(\dfrac{12}{5}\right)^2+\left(\dfrac{16}{5}\right)^2+\ldots.$ to ten terms

$$=\frac{4^2}{5^2}\left(2^2+3^2+4^2+\ldots.+11^2\right)=\frac{16}{25}\left(1^2+2^2+3^2+\ldots.+11^2-1^2\right)=\frac{16}{25}\left[\frac{11\times12\times23}{6}\right]=\frac{16}{25}\times505=\frac{16}{5}\times101$$

Given $\dfrac{16}{5}\times101=\dfrac{16}{5}m \Rightarrow m=101$

Trigonometric Identities & Equations

1.(B) $a\sin x + 1 - 2\sin^2 x = 2a-7 \Rightarrow 2\sin^2 x - a\sin x + 2a - 8 = 0 \Rightarrow \sin x = \dfrac{a \pm a - 8}{4}$

$$\Rightarrow \sin x = \frac{a-4}{2}, \quad [\text{Reject } \sin x = 2]$$

$$\Rightarrow \sin x = \frac{a-4}{2} \Rightarrow -1 \le \frac{a-4}{2} \le 1 \Rightarrow 2 \le a \le 6$$

2.(B) $\tan^2 x + \sec x = a \Rightarrow \sec^2 x + \sec x - 1 = a$

$$t^2+t-1=a$$

$$t \in (-\infty, -1] \cup [1, \infty)$$

Check from the graph to get : $a \in [-1, \infty)$

3.(A) $\sin(\theta - x) = a \qquad \cos(\theta - y) = b$

$\cos(\theta - x) = \sqrt{1-a^2} \qquad \sin(\theta - y) = \sqrt{1-b^2}$

$$\cos(x-y) = \cos((\theta-y)-(\theta-x)) = \cos(\theta-y)\cos(\theta-x)+\sin(\theta-x)\sin(\theta-y) = b\sqrt{1-a^2}+a\sqrt{1-b^2}$$

4.(D) $cot123° cot133° cot137° cot147°$

$= cot(90+33)cot(90+43)cot(180-43)cot(180-33) = (-tan33)(-tan43)(-cot43)(-cot33) = 1$

5.(B) $\text{LHS} = (1+tan1°)(1+tan44°)(1+tan2°)(1+tan43°).....(1+tan45°)$

$= (1+tan1°+tan44°+tan1° tan44°)+(1+tan2°+tan43°+tan2° tan43°).....(1+tan45°)$

$= (1+1)(1+1)....(1+1)23 \text{ times} = 2^{23}$

$n = 23$

6.(A) $4^{sin2x+2cos^2x} + 4^{1-sin2x+2sin^2x} = 65 \quad \Rightarrow \quad 4^{sin2x+cos2x+1} + 4^{1-sin2x-cos2x+1} = 65$

$4^{sin2x+cos2x+1} + 4^{3-(1+sin2x+cos2x)} = 65 \quad \Rightarrow \quad 4^{sin2x+cos2x+1} = y$

$y + \frac{64}{y} = 65 \quad \Rightarrow \quad y^2 - 65y + 64 = 0$

$y = 1, y = 64$

$4^{sin2x+cos2x+1} = 4^0$ or $4^{sin2x+cos2x+1} = 4^3$

$sin2x + cos2x = -1$

7.(C) $P = cos\frac{\pi}{20}cos\frac{3\pi}{20}cos\frac{7\pi}{20}cos\frac{9\pi}{20} \quad \Rightarrow \quad P = cos\frac{\pi}{20}sin\frac{\pi}{20}cos\frac{3\pi}{20}sin\frac{3\pi}{20}$

$P = \frac{1}{4}sin\frac{\pi}{10}sin\frac{3\pi}{10} = \frac{1}{4}\frac{\sqrt{5}-1}{4} \times \frac{\sqrt{5}+1}{4} = \frac{1}{16}$

$Q = cos\frac{\pi}{11}cos\frac{2\pi}{11}cos\frac{4\pi}{11}cos\frac{8\pi}{11}cos\frac{16\pi}{11}$; $Q = \frac{1}{2^5}\frac{sin2^5\frac{\pi}{11}}{sin\frac{\pi}{11}} = \frac{1}{32}\frac{sin\left(3\pi - \frac{\pi}{11}\right)}{sin\frac{\pi}{11}} = \frac{1}{32} \quad \Rightarrow \quad \frac{P}{Q} = \frac{\frac{1}{16}}{\frac{1}{32}} = 2$

8.(C) Given, $4sin^4x + cos^4x - 1 = 0$

$\Rightarrow \quad 4sin^4x + (cos^2x-1)(cos^2x+1) = 0 \quad \Rightarrow \quad 4sin^4x - sin^2x(1-sin^2x+1) = 0$

$\Rightarrow \quad sin^2x(5sin^2x-2) = 0 \quad \Rightarrow \quad sinx = 0 \text{ or } \pm\sqrt{\frac{2}{5}}$

Hence, $x = n\pi$ is the required solution.

9.(C) ABCD is a cyclic quadrilateral $\Rightarrow A + C = 180°, B + D = 180° \Rightarrow C = 180° - A, D = 180° - B$

$\Rightarrow \quad cos C = cos(180° - A) = -cos A, cos D = cos(180° - B) = -cos B$

$\Rightarrow \quad cos A + cos C = 0, cos B + cos D = 0 \Rightarrow cos A + cos B + cos C + cos D = 0$

10.(D) $3sinP + 4cosQ = 6$ ....(i) and $4sinQ + 3cosP = 1$ ....(ii)

Squaring and adding (i) and (ii), we get : $9 + 16 + 24(sinP cosQ + cosP sinQ) = 37$

$\Rightarrow \quad sin(P+Q) = \frac{1}{2} \Rightarrow sinR = \frac{1}{2} \Rightarrow R = \frac{\pi}{6}$

11.(A) Given that $tan\theta_1 . tan\theta_2 = k$ $\quad \frac{cos(\theta_1-\theta_2)}{cos(\theta_1+\theta_2)} = \frac{cos\theta_1 cos\theta_2 + sin\theta_1 sin\theta_2}{cos\theta_1 cos\theta_2 - sin\theta_1 sin\theta_2} = \frac{1+tan\theta_1 tan\theta_2}{1-tan\theta_1 tan\theta_2} = \frac{1+k}{1-k}$

12.(D) $\frac{1}{cos\alpha + cos(2n+1)\alpha} = \frac{1}{2cos(n+1)\alpha . cosn\alpha} = \frac{sin\alpha}{2sin\alpha . cos(n+1)\alpha . cosn\alpha}$

$= \frac{sin[(n+1)\alpha - n\alpha]}{2sin\alpha . cos(n+1)\alpha . cosn\alpha} = \frac{sin(n+1)\alpha . cosn\alpha - cos(n+1)\alpha . sinn\alpha}{2sin\alpha . cos(n+1)\alpha . cosn\alpha}$

$$= \frac{1}{2}\operatorname{cosec}\alpha\left[\frac{\sin(n+1)\alpha \,.\, \cos n\alpha}{\cos(n+1)\alpha \,.\, \cos n\alpha} - \frac{\cos(n+1)\alpha \,.\, \sin n\alpha}{\cos(n+1)\alpha \,.\, \cos n\alpha}\right] = \frac{1}{2}\operatorname{cosec}\alpha\left[\tan(n+1)\alpha - \tan n\alpha\right]$$

$$\therefore \qquad \frac{1}{\cos\alpha + \cos(2n+1)\alpha} = \frac{1}{2}\operatorname{cosec}\alpha\left[\tan(n+1)\alpha - \tan n\alpha\right]$$

Put $n = 1, 2, \ldots$ Then $\frac{1}{\cos\alpha + \cos 3\alpha} = \frac{1}{2}\operatorname{cosec}\alpha[\tan 2\alpha - \tan\alpha]$,

$$\frac{1}{\cos\alpha + \cos 5\alpha} = \frac{1}{2}\operatorname{cosec}\alpha[\tan 3\alpha - \tan 2\alpha] \ldots$$

$$\frac{1}{\cos\alpha + \cos 3\alpha} + \frac{1}{\cos\alpha + \cos 5\alpha} + \ldots + \frac{1}{\cos\alpha + \cos(2n+1)\alpha}$$

$$= \frac{1}{2}\operatorname{cosec}\alpha\left[(\tan 2\alpha - \tan\alpha) + (\tan 3\alpha - \tan 2\alpha) + \ldots + \{\tan(n+1)\alpha - \tan n\alpha\}\right]$$

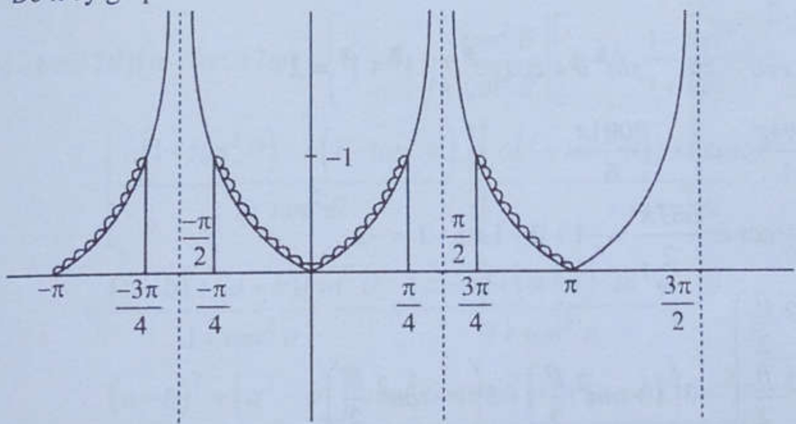
$$= \frac{1}{2}\operatorname{cosec}\alpha\left[\tan(n+1)\alpha - \tan\alpha\right]$$

13.(A) $-\sqrt{2} \le \sin x - \cos x \le \sqrt{2} \;\Rightarrow\; -2 \le \sqrt{2}(\sin x - \cos x) \le 2 \;\Rightarrow\; 1 \le \sqrt{2}(\sin x - \cos x) + 3 \le 5$

$\Rightarrow \qquad 0 \le \log_{\sqrt{5}}\left[\sqrt{2}(\sin x - \cos x) + 3\right] \le 2$

14.(C) $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15} = -\cos\left(\frac{\pi}{15}\right)\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right) = \frac{1}{16}$

15.(A) Do it by graph



16.(B) $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$

Let $x + \frac{1}{x} = t \qquad \Rightarrow \qquad \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) = \frac{1}{2}\left(t^2 - 2\right) = \frac{1}{2}\left((2\cos\theta)^2 - 2\right) = 2\cos^2\theta - 1 = \cos 2\theta$

17.(B) $\sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}} + \frac{\sqrt{1-\frac{b}{a}}}{\sqrt{1+\frac{b}{a}}} = \sqrt{\frac{1+\tan x}{1-\tan x}} + \frac{\sqrt{1-\tan x}}{\sqrt{1+\tan x}} = \frac{2}{\sqrt{1-\tan^2 x}} = \frac{2\sqrt{\cos^2 x}}{\sqrt{\cos^2 x - \sin^2 x}} = \frac{2\cos x}{\sqrt{\cos 2x}}$

18.(AD) $\sec\theta + \tan\theta = 1$

and $\sec^2\theta - \tan^2\theta = 1 \;\Rightarrow\; (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1 \Rightarrow \sec\theta - \tan\theta = 1 \;\Rightarrow\; \sec\theta = 1$ and $\tan\theta = 0$

Roots of the given equation are 1 and $\frac{c-2a+b}{a-2b+c} \qquad \Rightarrow \qquad \sec\theta$ and $\cos\theta$ are its roots ($\because$ both are 1).

19.(D) $\tan x + \cot x = 2$

We know that $\tan x + \cot x \le -2$ and $\tan x + \cot x \ge 2$

Limiting condition is when $\tan x = \cot x = 1$ $\therefore$ $x = \frac{\pi}{4}$ $\Rightarrow$ $\sin^{2n} x + \cos^{2n} x = \frac{1}{2^n} + \frac{1}{2^n} = \frac{1}{2^{n-1}}$

20.(D)

Function	Period
$\sin\frac{\pi}{2}x$	4
$\cos\frac{\pi}{3}x$	6
$\tan\frac{\pi}{3}x$	3

Take LCM of (4, 6, 3) = 12

21.(B) $P_n = \cos^n\theta + \sin^n\theta$

$P_n - P_{n-2} = \cos^n\theta + \sin^n\theta - \cos^{n-2}\theta - \sin^{n-2}\theta$

$= \cos^{n-2}\theta\left(\cos^2\theta - 1\right) + \sin^{n-2}\theta\left(\sin^2\theta - 1\right) = \cos^{n-2}\theta\left(-\sin^2\theta\right) + \sin^{n-2}\theta\left(-\cos^2\theta\right)$

$= -\sin^2\theta\cos^2\theta\left(\cos^{n-4}\theta + \sin^{n-4}\theta\right) = -\sin^2\theta\cos^2\theta\ P_{n-4}$

22.(A) $\sin\theta + \operatorname{cosec}\theta \ge 2$ and $\sin\theta + \operatorname{cosec}\theta \le -2$

For $\sin\theta + \operatorname{cosec}\theta = 2$; $\sin\theta = 1 = \operatorname{cosec}\theta$ $\Rightarrow$ $\sin^8\theta + \operatorname{cosec}^8\theta = 1^8 + 1^8 = 2$

23.(C) $\cos 2001\pi + \cot 2001\frac{\pi}{2} + \sec\frac{2001\pi}{3} + \tan\frac{2001\pi}{4} + \operatorname{cosec}\frac{2001\pi}{6}$

$= \cos\pi + \cot\frac{\pi}{2} + \sec 667\pi + \tan\frac{\pi}{4} + \operatorname{cosec}\frac{667\pi}{2} = -1 + 0 - 1 + 1 - 1 = -2$

24.(A) $2\tan\frac{\alpha}{2} = \tan\frac{\beta}{2}$; $\frac{3+5\cos\beta}{5+3\cos\beta} = \frac{3+5\left(\frac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}\right)}{5+3\left(\frac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}\right)} = \frac{3\left(1+\tan^2\frac{\beta}{2}\right)+5\left(1-\tan^2\frac{\beta}{2}\right)}{5\left(1+\tan^2\frac{\beta}{2}\right)+3\left(1-\tan^2\frac{\beta}{2}\right)}$

$= \frac{8-2\tan^2\frac{\beta}{2}}{8+2\tan^2\frac{\beta}{2}} = \frac{8-8\tan^2\frac{\alpha}{2}}{8+8\tan^2\frac{\alpha}{2}} = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \cos\alpha$

25.(B) $\left(2\cos^2 x\right)^2 - 2\cos 2x - \frac{1}{2}\cos 4x$

$= \left(1+\cos 2x\right)^2 - 2\cos 2x - \frac{1}{2}\left(2\cos^2 2x - 1\right) = 1 + \cos^2 2x + 2\cos 2x - 2\cos 2x - \cos^2 2x + \frac{1}{2} = \frac{3}{2}$

26.(AD) $\sin\theta + \sin\phi = a$ ....(i) & $\cos\theta + \cos\phi = b$ ....(ii)

Squaring and adding

$$\sin^2\theta+\sin^2\phi+2\sin\theta\sin\theta=a^2 \quad\Rightarrow\quad \cos^2\theta+\cos^2\phi+2\cos\theta\cos\phi=b^2$$

$$2+2(\cos\theta\cos\phi+\sin\phi\sin\theta)=a^2+b^2 \Rightarrow \cos(\theta-\phi)=\frac{a^2+b^2-2}{2}$$

$$2\cos^2\left(\frac{\theta-\phi}{2}\right)-1=\frac{a^2+b^2}{2}-1 \Rightarrow \cos^2\left(\frac{\theta-\phi}{2}\right)=\frac{a^2+b^2}{4} \Rightarrow \cos\left(\frac{\theta-\phi}{2}\right)=\pm\frac{1}{2}\sqrt{a^2+b^2}$$

27.(BD) $0<x<\pi$

$0<y<\pi$

$-\pi<-y<0$

$-\pi<x-y<\pi$

$0<x+y<2\pi$

$\sin(x-y)=\frac{1}{2}$

$x-y=\frac{\pi}{6}$ and $x-y=\frac{5\pi}{6}$

$x+y=\frac{\pi}{3}$ and $x+y=\frac{5\pi}{3}$

$x=\frac{\pi}{4}$ $\quad y=\frac{\pi}{12}$

$x=\frac{11\pi}{12}$ $\quad y=\frac{3\pi}{4}$

28.(D) $$(a-b\cos 2\theta)(a-b\cos 2\varphi)=\left[a-b\cdot\frac{1-\tan^2\theta}{1+\tan^2\theta}\right]\left[a-b\cdot\frac{1-\tan^2\varphi}{1+\tan^2\varphi}\right]$$

$$=\left[\frac{a(1+\tan^2\theta)-b(1-\tan^2\theta)}{1+\tan^2\theta}\right]\left[\frac{a(1+\tan^2\varphi)-b(1-\tan^2\varphi)}{1+\tan^2\varphi}\right]$$

$$=\frac{(a-b)+(a+b)\tan^2\theta}{1+\tan^2\theta}\times\frac{(a-b)+(a+b)\tan^2\varphi}{1+\tan^2\varphi}$$

$$=\frac{(a-b)^2+(a^2-b^2)\left[\tan^2\theta+\tan^2\varphi\right]+(a+b)^2\tan^2\theta\tan^2\varphi}{1+(\tan^2\theta+\tan^2\varphi)+\tan^2\theta\,.\,\tan^2\varphi}$$

$$=\frac{(a-b)^2+(a^2-b^2)(\tan^2\theta+\tan^2\varphi)+(a+b)^2\left(\frac{a-b}{a+b}\right)}{1+(\tan^2\theta+\tan^2\varphi)+\frac{a-b}{a+b}}=\frac{(a-b)\left[a-b+(a+b)(\tan^2\theta+\tan^2\varphi)+a+b\right]}{\left[\frac{a+b+(a+b)(\tan^2\theta+\tan^2\varphi)+a-b}{(a+b)}\right]}$$

$$=\frac{(a+b)(a-b)\left[2a+(a+b)(\tan^2\theta+\tan^2\varphi)\right]}{\left[2a+(a+b)(\tan^2\theta+\tan^2\varphi)\right]}=a^2-b^2$$

29.(C) $\sin x+\cos x+\tan x+\cot x+\sec x+\operatorname{cosec} x=7$

$$\Rightarrow\quad (\sin x+\cos x)+\left(\frac{1}{\sin x\,.\,\cos x}\right)+\frac{(\sin x+\cos x)}{\sin x\,.\,\cos x}=7 \Rightarrow (\sin x+\cos x)\left(1+\frac{2}{\sin 2x}\right)=\left(7-\frac{2}{\sin 2x}\right)$$

Squaring, we have $(1+\sin 2x)\left(1+\frac{4}{\sin^2 2x}+\frac{4}{\sin 2x}\right)=49+\frac{4}{\sin^2 2x}-\frac{28}{\sin 2x}$

$\Rightarrow \quad 1+\frac{4}{\sin^2 2x}+\frac{4}{\sin 2x}+\sin 2x+\frac{4}{\sin 2x}+4=49+\frac{4}{\sin^2 2x}-\frac{28}{\sin 2x} \Rightarrow \sin 2x-44+\frac{36}{\sin 2x}=0$

$\Rightarrow \quad \sin^2 2x-44\sin 2x+36=0 \Rightarrow \sin 2x=\frac{44\pm\sqrt{1936-144}}{2}=\frac{44\pm\sqrt{1792}}{2}=22\pm 8\sqrt{7}$

$\sin 2x \le 1 \Rightarrow \sin 2x = 22-8\sqrt{7} \quad \therefore \quad a=22, b=8$

30.(A) Sum of the roots $=\sin^2 18°+\cos^2 36°=\left[\frac{\sqrt{5}-1}{4}\right]^2+\left[\frac{\sqrt{5}+1}{4}\right]^2=\frac{1}{16}\left[\left(\sqrt{5}+1\right)^2+\left(\sqrt{5}-1\right)^2\right]=\left(\frac{1}{16}\right)\left[2(5+1)\right]=\frac{3}{4}$

Product of the roots $=\sin^2 18°\cos^2 36°=\left[\frac{\sqrt{5}-1}{4}\right]^2\left[\frac{\sqrt{5}+1}{4}\right]^2=\left[\frac{5-1}{4\,.\,4}\right]^2=\frac{1}{16}$

$\Rightarrow$ Required quadratic equation is $x^2-\frac{3x}{4}+\frac{1}{16}=0$ or $16x^2-12x+1=0$

31.(C) Expanding D, we get $D=1+\sin\theta\cos\theta-\cos\theta(-\sin\theta-\cos\theta)+\left(-\sin^2\theta+1\right)$

$=2+2\sin\theta\cos\theta+\cos 2\theta=2+\sin 2\theta+\cos 2\theta=2+\sqrt{2}\cos\left(2\theta-\frac{\pi}{4}\right)$

As $\quad -1\le\cos\left(2\theta-\frac{\pi}{4}\right)\le 1,\ 2-\sqrt{2}\le D\le 2+\sqrt{2}$

32.(C) From the given relation we have $\sin x-\sin 3x+2\sin 2x=3 \Rightarrow 2\sin x\cos 2x-2\sin 2x+3=0$

$\Rightarrow \quad (\sin x+\cos 2x)^2+(\sin 2x-1)^2+3=\sin^2 x+\cos^2 2x+\sin^2 2x+1$

$\Rightarrow \quad (\sin x+\cos 2x)^2+(\sin 2x-1)^2+\cos^2 x=0$ which is possible only if

$\sin x+\cos 2x=0,\ \sin 2x=1$ and $\cos x=0$ which is not possible for any value of x.

33.(D) $(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2=7+\tan^2\theta+\cot^2\theta \quad \therefore$ Minimum value = 9

34.(B) $\frac{1}{2}\tan\frac{\alpha}{2}+\frac{1}{4}\tan\frac{\alpha}{4}+\frac{1}{8}\tan\frac{\alpha}{8}\ldots\ldots\frac{1}{2^n}\tan\frac{\alpha}{2^n}+\cot\alpha$

$\cot\alpha+\frac{1}{2}\tan\frac{\alpha}{2}=\frac{1-\tan^2\frac{\alpha}{2}}{2\tan\frac{\alpha}{2}}+\frac{1}{2}\tan\frac{\alpha}{2}=\frac{1}{2\tan\frac{\alpha}{2}}=\frac{1}{2}\cot\frac{\alpha}{2}$

Similarly $\frac{1}{4}\tan\frac{\alpha}{4}+\frac{1}{2}\cot\frac{\alpha}{2}=\frac{\cot\frac{\alpha}{4}}{4} \quad \therefore$ equation reduces to $\frac{\cot\frac{\alpha}{2^n}}{2^n}$

35.(B) $25\cos^2\theta+5\cos\theta-12=0$

$\Rightarrow \quad \cos\theta=\frac{-5\pm\sqrt{25+4\times 25\times 12}}{50}=\frac{-5\pm 35}{50}=\frac{-4}{5},\frac{3}{5}$

$\cos\theta=\frac{-4}{5}$ as $\frac{\pi}{2}<\alpha<\pi \quad \Rightarrow \quad \sin\theta=\frac{3}{5} \quad \Rightarrow \quad \sin 2\theta=2\sin\theta\cos\theta=\frac{-24}{25}$

36.(B) $x=\frac{-y}{2}=\frac{-z}{2}=k\ ; \qquad xy+yz+zx=-2k^2+4k^2+\left(-2k^2\right)=0$

37.(B) $\cos\left(x-\frac{3\pi}{2}\right)+\sin\left(\frac{3\pi}{2}+x\right)+\sin(32\pi+x)-18\cos(19\pi-x)+\cos(56\pi+x)-9\sin(x+17\pi)$

$=-\sin x-\cos x+\sin x+18\cos x+\cos x+9\sin x=18\cos x+9\sin x$

$a+b=27$

38.(B) $\left[(\cos\beta\cos\gamma+\sin\beta\sin\gamma)+(\cos\gamma\cos\alpha+\sin\alpha\sin\gamma)+\cos\alpha\cos\beta+\sin\alpha\sin\beta\right]=-\frac{3}{2}$

$1+1+1+2\left[\sum\cos\alpha\cos\beta+\sum\sin\alpha\sin\beta\right]=0$

$\cos^2\alpha+\sin^2\alpha+\cos^2\beta+\sin^2\beta+\cos^2\gamma+\sin^2\gamma+2\left[\sum\cos\alpha\cos\beta+\sum\sin\alpha\sin\beta\right]=0$

$(\cos\alpha+\cos\beta+\cos\gamma)^2+(\sin\alpha+\sin\beta+\sin\gamma)^2=0$

$\Rightarrow \quad \cos\alpha+\cos\beta+\cos\gamma=0 \quad$ and $\quad \sin\alpha+\sin\beta+\sin\gamma=0$

39.(D) $\left|\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}\right|=1$

$\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}=\pm1$

$\sqrt{2\sin^4 x+18\cos^2 x}=\sqrt{2\cos^4 x+18\sin^2 x}\pm1$

Squaring on both sides $2\sin^4 x+18\cos^2 x=2\cos^4 x+18\sin^2 x+1\pm2\sqrt{2\cos^4 x+18\sin^2 x}$

$\Rightarrow \quad 2\left(\sin^4 x-\cos^4 x\right)+18\left(\cos^2 x-\sin^2 x\right)-1=\pm2\sqrt{2\cos^4 x+18\sin^2 x}$

$\Rightarrow \quad 16\cos 2x-1=\pm2\sqrt{2\cos^4 x+18\sin^2 x}$

$\Rightarrow \quad (16\cos 2x-1)^2=4\left[2\left(\frac{1+\cos 2x}{2}\right)^2+9(1-\cos 2x)\right]$

$\Rightarrow \quad \cos^2 2x=\frac{37}{254} \quad \Rightarrow \quad \cos 2x=\pm\sqrt{\frac{37}{254}}$.

But $x\in[0,2\pi] \quad \Rightarrow \quad 2x\in[0,4\pi]$.

Clearly, the number of solutions is 8.

40.(B) $f'(x)=4\sin^3 x\cos x-4\cos^3 x\sin x=4\sin x\cos x\left(\sin^2 x-\cos^2 x\right)=2\sin 2x\cos 2x=4\sin 4x$

For increasing function, $f'(x)>0$

$\therefore \quad -\sin 4x>0 \quad \Rightarrow \quad \sin 4x<0 \quad \Rightarrow \quad \pi<4x<2\pi \quad \Rightarrow \quad \frac{\pi}{2}<x<\frac{\pi}{2}$

$\therefore \quad$ Increasing in $\left]\frac{\pi}{4},\frac{\pi}{2}\right[$

41.(C) $f(A)=\tan A+\tan B$

$f(A)=\tan A+\tan\left(\frac{\pi}{6}-A\right) \qquad \left\{\because A+B=\frac{\pi}{6}\right\}$

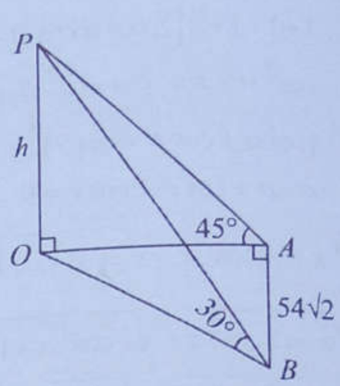
$f'(A)=\sec^2 A-\sec^2\left(\frac{\pi}{6}-A\right)$

$f'(A) = 0 \quad \Rightarrow \quad \sec^2 A = \sec^2\left(\frac{\pi}{6} - A\right) \quad \Rightarrow \quad A = \frac{\pi}{6} - A \quad \Rightarrow \quad A = \frac{\pi}{12}$

$f''(A) > 0$ at $A = \frac{\pi}{12} = 15°$

Min. value $= \tan 15° + \tan 15° = 2\left(2 - \sqrt{3}\right) = 4 - 2\sqrt{3}$

42.(B) Let OP be a tower, due east of tower is A and due south of A is B.



In ΔPOA, $\tan 45° = h / OA \quad \Rightarrow \quad OA = h$

In ΔPOB, $PO \perp OB$

$\tan 30° = h / OB \quad \Rightarrow \quad OB = \sqrt{3}h$

In ΔAOB, $OB^2 = OA^2 + AB^2$

$\Rightarrow \quad \left(\sqrt{3}h\right)^2 = h^2 + \left(54\sqrt{2}\right)^2 \quad \left\{\because AB = 54\sqrt{2}\right\} \quad \Rightarrow \quad h = 54$

43.(B) In ΔABC, $\angle C = 60° \quad \Rightarrow \quad A + B = 120°$

Sine formula $\frac{a}{\sin A} = \frac{b}{\sin B} \quad \Rightarrow \quad \frac{a}{b} = \frac{\sin A}{\sin B} = \frac{\sin(120° - B)}{\sin B}$

$2 + \sqrt{3} = \frac{\sin 120° \cos B - \cos 120° \sin B}{\sin B} \quad \Rightarrow \quad 2 + \sqrt{3} = \frac{\sqrt{3}}{2}\cot B + \frac{1}{2}$

$\cot B = 2 + \sqrt{3} = \cot 15° \quad \therefore \quad \angle B = 15°$ and $\angle A = 105°$

44.(B) $2\tan^{-1} x = \begin{cases} \sin^{-1}\frac{2x}{1+x^2}, & |x| < 1 \\ \pi - \sin^{-1}\frac{2x}{1+x^2}, & x > 1 \end{cases}$

$\therefore \quad f(x) = 2\tan^{-1} x + \sin^{-1}\frac{2x}{1+x^2} = \pi - \sin^{-1}\frac{2x}{1+x^2} + \sin^{-1}\frac{2x}{1+x^2} = \pi \quad \therefore \quad f(5) = \pi$

45.(A) A stone is thrown upwards vertically with the velocity of $48\, m/s$, so $u = 48\, m/s$.

When the stone attains its greatest height, then $v = 0$.

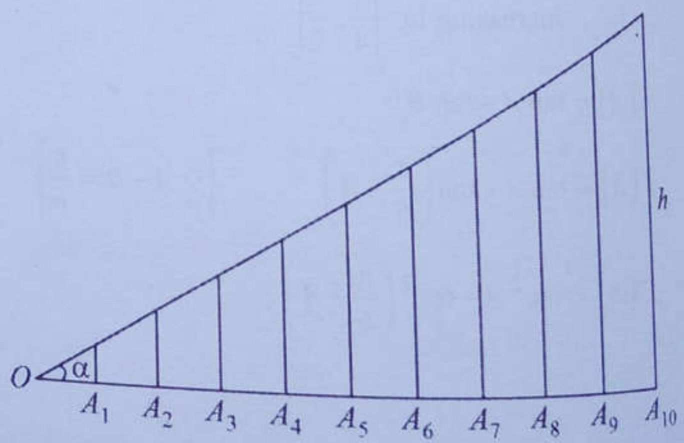
Now, $v^2 = u^2 - 2gs$

$o = 48^2 - 2 \times 32 \times s$

$s = 36\, m$

Since the stone is thrown from the top of a $64\, m$ high tower, so the greatest height obtained by the stone $= 64 + 36 = 100\, m$

46.(C) $OA_1 = a$, $A_1 A_2 = A_2 A_3 = ... = A_9 A_{10} = x$ (let)



$\therefore \quad A_1 A_{10} = 9x$ and $OA_{10} = a + 9x$

$\therefore \quad \tan\alpha = \frac{h}{OA_{10}} = \frac{h}{a + 9x}$

$\Rightarrow \quad a + 9x = h\cot\alpha$

$\Rightarrow \quad x = \frac{h\cot\alpha - a}{9} = \frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$

47.(C) $f(x) = 3\sin x + 4\cos x$ for $x \geq 0$

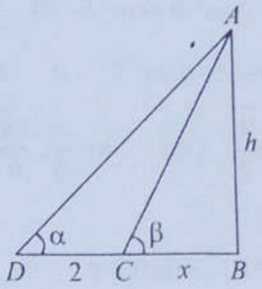
But $f(x)$ is an odd function $\therefore$ $f\left(-\frac{11\pi}{6}\right) = -f\left(\frac{11\pi}{6}\right) = -\left[3\sin\frac{11\pi}{6} + 4\cos\frac{11\pi}{6}\right] = -\left[-\frac{3}{2} + 4\cdot\frac{\sqrt{3}}{2}\right] = \frac{3}{2} - 2\sqrt{3}$

48.(A) In ΔABC : $h = x\tan B$

In ΔABD : $h = (x+2)\tan\alpha$

$h = \left(\frac{h}{\tan\beta} + 2\right)\tan\alpha \quad \Rightarrow \quad h\left(1 - \frac{\tan\alpha}{\tan\beta}\right) = 2\tan\alpha$

$\Rightarrow \quad h = \frac{2\tan\alpha\tan\beta}{\tan\beta - \tan\alpha} \Rightarrow \quad h = \frac{2\sin\alpha\sin\alpha}{\sin(\beta-\alpha)}$



49.(A) $f(\theta) = 1 + \sin 2\theta + 2\cos^2\theta = 2 + \sin 2\theta + \cos 2\theta$

But $-\sqrt{2} \leq \sin 2\theta + \cos 2\theta \leq 2 + \sqrt{2}$

$\Rightarrow \quad 2 - \sqrt{2} \leq 2 + \sin 2\theta + \cos 2\theta \leq 2 + \sqrt{2} \quad \Rightarrow \quad 2 - \sqrt{2} \leq f(\theta) \leq 2 + \sqrt{2}$

$\therefore \quad A = \text{max. value} = 2 + \sqrt{2}$

$B = \text{min. value} = 2 - \sqrt{2}$

50.(A) **Statement I :** $-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}$ and $0 \leq \cos^{-1}x \leq \pi$ $\quad\therefore\quad$ $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ has some finite value.

Obviously, $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 - a\pi^3 = 0$ has not a solution for all $a \geq \frac{1}{32}$.

For example, if $a = 100$, then $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 \neq 100\pi^3$ for any x. $\quad\therefore\quad$ Statement I is false

Statement II : $0 \leq \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 \leq \frac{9\pi^2}{16}$

$\left|\sin^{-1}x - \frac{\pi}{4}\right| \leq \frac{3\pi}{4}$

$-\frac{3\pi}{4} \leq \sin^{-1}x - \frac{\pi}{4} \leq \frac{3\pi}{4}$

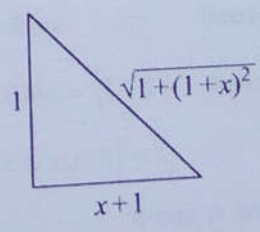
$-\frac{\pi}{2} \leq \sin^{-1}x \leq \pi$ is not true because $-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}$ $\quad\therefore\quad$ Statement II is false

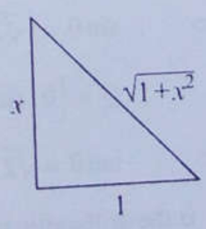
51.(C) Period of $|\sin x| = \pi$ and period of $|\cos x| = \pi$ $\quad\therefore\quad$ Period of $|\sin 4x| = \pi/4$ and period of $|\cos 2x| = \pi/2$

LMC of $\pi/4$ and $\pi/2 = \pi/2$ $\quad\therefore\quad$ Period of $f(x) = \pi/2$

52.(C) $\tan^{-1}\left(\cot\left(\frac{40\pi + 3\pi}{4}\right)\right) = \tan^{-1}\left(\cot\left(10\pi + \frac{3\pi}{4}\right)\right) = \tan^{-1}\left(\cot\frac{3\pi}{4}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)\right) = \tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right) = -\frac{\pi}{4}$

53.(A) $\sin\left(\cot^{-1}(1+x)\right) = \cos\left(\tan^{-1}x\right)$





$\Rightarrow \quad \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(1+x)^2}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$

$\Rightarrow \quad \frac{1}{\sqrt{1+(1+x)^2}} = \frac{1}{\sqrt{1+x^2}} \quad \Rightarrow \quad 1 + (1+x)^2 = 1 + x^2 \quad \Rightarrow \quad x = -1/2$

54.(D) $\sin^{-1}x = 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} \Rightarrow x = \frac{2x}{1+x^2} \Rightarrow x = 0, 1, -1$

All the three values of x satisfy the equation.

55.(B) **Statement 1 :** $2\sin^2\theta - \cos^2\theta = 0$

$\Rightarrow 2\sin^{-2}\theta - \left(1-2\sin^2\theta\right) = 0 \Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \pm\frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$2\cos^2\theta - 3\sin\theta = 0$

$\Rightarrow 2\left(1-\sin^2\theta\right) - 3\sin\theta = 0 \Rightarrow (2\sin\theta - 1)(\sin\theta + 2) = 0$

$\Rightarrow \sin\theta = 1/2$ as $\sin\theta + 2 \neq 0 \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ in $[0, 2\pi]$

$\therefore$ Number of common solutions is 2.

Statement 2 : $2\cos^2\theta - 3\sin\theta = 0$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ in $[0, \pi]$

Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

56.(C) $\tan^{-1}\left(\frac{1}{n^2+n+1}\right) = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right) = \tan^{-1}(n+1) - \tan^{-1}n, \tan^{-1}\left(\frac{1}{n^2+3n+3}\right)$

$= \tan^{-1}\left(\frac{(n+2)-(n+1)}{1+(n+2)(n+1)}\right) = \tan^{-1}(n+2) - \tan^{-1}(n+1), \ldots, \tan^{-1}\left(\frac{1}{1+(n+19)(n+20)}\right)$

$= \tan^{-1}(n+20) - \tan^{-1}(n+19)$

$\therefore S = \tan^{-1}(n+20) - \tan^{-1}n = \tan^{-1}\left(\frac{n+20-n}{1+n(n+20)}\right) = \tan^{-1}\left(\frac{20}{n^2+20n+1}\right) \Rightarrow \tan S = \frac{20}{n^2+20n+1}$

57.(B) $4 + \frac{1}{2}\sin^2 2x - 2\left(\cos^2 x\right)^2 = 4 + \frac{1}{2}\sin^2 2x - 2\left(\frac{1+\cos 2x}{2}\right)^2 = 4 + \frac{1}{2}\left(1-\cos^2 2x\right) - \frac{1}{2}\left(1 + 2\cos 2x + \cos^2 2x\right)$

$= 4 - \cos^2 2x - \cos 2x = \frac{17}{4} - \left(\cos 2x + \frac{1}{2}\right)^2 \quad \therefore$ Max. value $= M = \frac{17}{4}$, when $\cos 2x + \frac{1}{2} = 0$.

Min. value $= m = \frac{17}{4} - \left(1+\frac{1}{2}\right)^2 = 2$, when $\cos 2x = 1 \quad \therefore \quad M - m = \frac{17}{4} - 2 = \frac{9}{4}$.

58.(D) $P = \left\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\right\} = \sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \left(\sqrt{2}+1\right)\cos\theta \Rightarrow \tan\theta = \sqrt{2}+1 \quad \therefore \quad P = \left\{\theta : \tan\theta = \sqrt{2}+1\right\}$

$Q = \left\{\theta : \sin\theta + \cos\theta = \sqrt{2}+1\right\} = \sin\theta + \cos\theta = \sqrt{2}\sin\theta = \cos\theta = \left(\sqrt{2}-1\right)\sin\theta$

$\Rightarrow \tan\theta = \sqrt{2}+1 \quad \therefore \quad Q = \left\{\theta : \tan\theta = \sqrt{2}+1\right\}$ Hence, $P = Q$.

59.(C) θ is the arithmetic mean of α and β

$\therefore \theta = \frac{\alpha+\beta}{2}$

Now, $\cos\alpha + \cos\beta = \frac{3}{2}$ $\Rightarrow$ $2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{3}{2}$ ...... (i)

and $\sin\alpha + \sin\beta = \frac{1}{2}$ $\Rightarrow$ $2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{1}{2}$ ...... (ii)

Dividing (ii) by (i), we get : $\tan\frac{\alpha+\beta}{2} = \frac{1}{3}$ $\Rightarrow$ $\tan\theta = \frac{1}{3}$

$$\therefore \quad \sin 2\theta + \cos 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} + \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{\frac{2}{3}}{1+\frac{1}{9}} + \frac{1-\frac{1}{9}}{1+\frac{1}{9}} = \frac{2}{3}\times\frac{9}{10} + \frac{8}{9}\times\frac{9}{10} = \frac{3}{5}+\frac{4}{5} = \frac{7}{5}$$

60.(C) $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha - 2 = 0$

$(\sin\alpha - 1)(2\sin^2\alpha - 5\sin\alpha + 2) = 0$

$(\sin\alpha - 1)(2\sin\alpha - 1)(\sin\alpha - 2) = 0$

$\sin\alpha = 1, \sin\alpha = 1/2, \sin\alpha \neq 2$

$a = \frac{\pi}{2}, \alpha = \frac{\pi}{6}, \frac{5\pi}{6}$ in $[0, 2\pi]$

$\therefore$ Number of values of α is 3

61.(B) $\text{cosec}\,\theta = \frac{p+q}{p-q}$ $\Rightarrow$ $\sin\theta = \frac{p-q}{p+q}$

$$\frac{1+\cos\theta}{1-\cos\theta} = \frac{2\cos^2\theta/2}{2\sin^2\theta/2} = \cot^2\frac{\theta}{2}$$

$$\therefore \quad \cot^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{1+\cos\left(\frac{\pi}{2}+\theta\right)}{1-\cos\left(\frac{\pi}{2}+\theta\right)} = \frac{1-\sin\theta}{1+\sin\theta} = \frac{1-\frac{p-q}{p+q}}{1+\frac{p-q}{p+q}} = \frac{q}{p} \quad \therefore \quad \left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right| = \sqrt{\frac{q}{p}}$$

62.(B) $2\cos\theta + \sin\theta = 1$ ...... (i)

$(2\cos\theta)^2 = (1-\sin\theta)^2$ $\Rightarrow$ $4(1-\sin^2\theta) = 1 - 2\sin\theta + \sin^2\theta$

$5\sin^2\theta - 2\sin\theta - 3 = 0$ $\Rightarrow$ $(\sin\theta - 1)(5\sin\theta + 3) = 0$

$\sin\theta = -\frac{3}{5}$ $\therefore$ $\theta \neq \frac{\pi}{2}$

From (i), $\cos\theta = \frac{4}{5}$ $\therefore$ $7\cos\theta + 6\sin\theta = 7\cdot\frac{4}{5} + 6\cdot\left(-\frac{3}{5}\right) = 2$

63.(A) $\sin 2x - 2\cos x + 4\sin x = 4$

$\Rightarrow$ $2\sin x\cos x - 2\cos x + 4\sin x = 4$ $\Rightarrow$ $2\cos x(\sin x - 1) + 4(\sin x - 1) = 0$

$\Rightarrow$ $(\sin x - 1)(2\cos x + 4) = 0$ $\Rightarrow$ $\sin x = 1$ as $\cos x + 2 \neq 0$

$\therefore$ $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$ in $[0, 5\pi]$