WBJEE - 2017

Answer Keys by

Aakash Institute, Kolkata Centre

MATHEMATICS

Q.No.	✚	☽	✿	☐
01	B	A	C	B
02	A	C	A	B
03	D	C	B	B
04	B	C	D	D
05	D	A	B	B
06	C	D	B	B
07	B	C	C	A
08	B	B	A	A
09	A	*	B	D
10	C	C	B	B
11	D	A	A	D
12	B	B	C	B
13	A	D	D	D
14	C	B	A	A
15	C	B	B	B
16	C	C	B	A
17	A	A	B	D
18	D	B	B	B
19	C	B	D	D
20	B	A	B	C
21	*	C	B	B
22	C	D	A	B
23	A	A	A	A
24	B	B	D	C
25	D	B	B	D
26	B	B	D	B
27	B	B	B	A
28	C	D	D	C
29	A	B	A	C
30	B	B	B	C
31	B	A	A	A
32	A	A	D	D
33	C	D	B	C
34	D	B	D	B
35	A	D	C	*
36	B	B	B	C
37	B	D	B	A
38	B	A	A	B
39	B	B	C	D
40	D	A	D	B
41	B	D	B	B
42	B	B	A	C
43	A	D	C	A
44	A	C	C	B
45	D	B	C	B
46	B	B	A	A
47	D	A	D	C
48	B	C	C	D
49	D	D	B	A
50	A	B	*	B
51	C	C	B	B
52	A	B	C	C
53	D	B	A	C
54	C	A	C	A
55	B	A	B	D
56	B	C	C	C
57	A	B	C	B
58	A	C	A	B
59	C	A	D	A
60	B	C	C	A
61	C	B	B	C
62	A	C	B	B
63	C	C	A	C
64	B	A	A	A
65	C	D	C	C
66	B	C	B,C	C,D
67	B,C	B,D	A,C	A,C
68	C	B,C	C	C
69	B,D	A,C	C,D	B
70	B,C	C	A,C	B,C
71	A,C	C,D	C	C
72	C	A,C	B	B,D
73	C,D	C	B,C	B,C
74	A,C	B	C	A,C
75	C	B,C	B,D	C

***** Either B or D.

Code - ✚

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ANSWERS & HINT

for

WBJEE - 2017

SUB : MATHEMATICS

CATEGORY - I (Q1 to Q50)

Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch $-\frac{1}{4}$ marks. No answer will fetch 0 marks.

1. Transforming to parallel axes through a point (p, q), the equation

   $2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ becomes $2x^2 + 3xy + 4y^2 = 1$. Then

   (A) p = –2, q = 3 (B) p = 2, q = – 3 (C) p = 3, q = – 4 (D) p = – 4, q = 3

   Ans : (B)

   Hint : $4p + 3q + 1 = 0$

   $3p + 8q + 18 = 0$

   $\therefore p = 2, q = -3$

2. Let A(2, –3) and B (–2, 1) be two angular points of ΔABC. If the centroid of the triangle moves on the line 2x + 3y = 1, then the locus of the angular point C is given by

   (A) 2x + 3y = 9 (B) 2x – 3y = 9 (C) 3x + 2y = 5 (D) 3x – 2y = 3

   Ans : (A)

   Hint : $G\left(t, \frac{1-2t}{3}\right), \alpha = 3t$

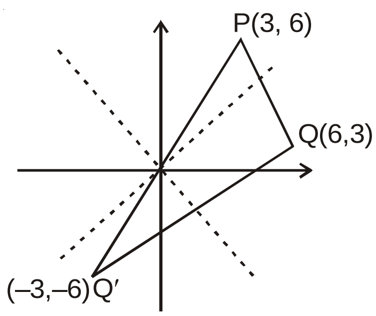
   $\beta = 3 - 2t, \therefore 2x + 3y = 9$

3. The point P (3, 6) is first reflected on the line y = x and then the image point Q is again reflected on the line y = – x to get the image point Q′. Then the circumcentre of the ΔPQQ′ is

   (A) (6, 3) (B) (6, – 3) (C) (3, –6) (D) (0, 0)

   Ans : (D)

   Hint :



4. Let d_1 and d_2 be the lengths of the perpendiculars drawn from any point of the line $7x - 9y + 10 = 0$ upon the lines $3x + 4y = 5$ and $12x + 5y = 7$ respectively. Then

(A) $d_1 > d_2$ (B) $d_1 = d_2$ (C) $d_1 < d_2$ (D) $d_1 = 2d_2$

Ans : (B)

Hint :

5. The common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $2x^2 + 2y^2 = 32$ subtends at the origin an angle equal to

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

Ans : (D)

Hint : This common chord is passing through the centre of the 1st circle. Therefore it will form an angle of 90º at the circumferential point (0, 0).

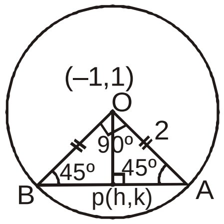
6. The locus of the mid-points of the chords of the circle $x^2 + y^2 + 2x - 2y - 2 = 0$ which make an angle of 90º at the centre is

(A) $x^2 + y^2 - 2x - 2y = 0$ (B) $x^2 + y^2 - 2x + 2y = 0$ (C) $x^2 + y^2 + 2x - 2y = 0$ (D) $x^2 + y^2 + 2x - 2y - 1=0$

Ans : (C)

$\sin 45^\circ = \frac{OP}{2} \Rightarrow OP = \sqrt{2}$, Centre : (–1, 1)

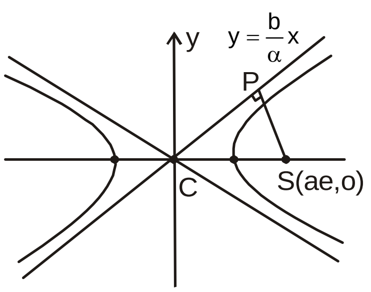
Hint :



7. Let P be the foot of the perpendicular from focus S of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ on the line $bx - ay = 0$ and let C be the centre of hyperbola. Then the area of the rectangle whose sides are equal to that of SP and CP is

(A) 2ab (B) ab (C) $\frac{(a^2 + b^2)}{2}$ (D) $\frac{a}{b}$

Ans : (B)



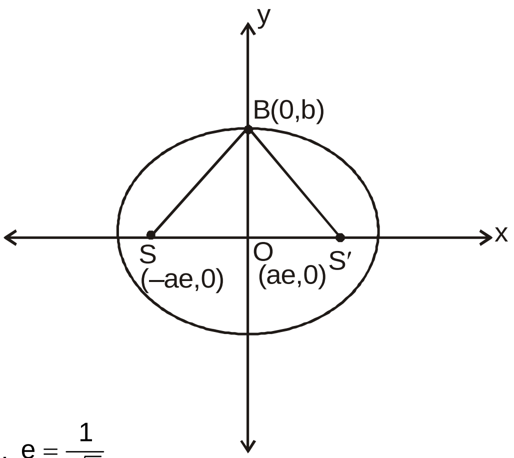
Hint : Area = SP.CP = a.b

8. B is an extremity of the minor axis of an ellipse whose foci are S and S′. If $\angle SBS'$ is a right angle, then the eccentricity of the ellipse is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Ans : (B)

Hint :



$b = ae,\ e = \frac{1}{\sqrt{2}}$

9. The axis of the parabola $x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$ is

(A) $x + y = 0$ (B) $x + y - 1 = 0$ (C) $x - y + 1 = 0$ (D) $x - y = \frac{1}{\sqrt{2}}$

Ans : (A)

Hint : $(x + y)^2 = 5x - 5y + 5 \Rightarrow (x + y)^2 = 5(x - y + 1)$

$\therefore$ Axis is $x + y = 0$

10. The line segment joining the foci of the hyperbola $x^2 - y^2 + 1 = 0$ is one of the diameters of a circle. The equation of the circle is

(A) $x^2 + y^2 = 4$ (B) $x^2 + y^2 = \sqrt{2}$ (C) $x^2 + y^2 = 2$ (D) $x^2 + y^2 = 2\sqrt{2}$

Ans : (C)

Hint : $\therefore x^2 - y^2 + 1 = 0$, Foci $= \left(0, \pm\sqrt{2}\right)$, Centre = (0, 0), Radius $= \sqrt{2}$

Equation of circle $x^2 + y^2 = 2$

11. The equation of the plane through (1, 2, –3) and (2, –2, 1) and parallel to X-axis is

(A) $y - z + 1 = 0$ (B) $y - z - 1 = 0$ (C) $y + z - 1 = 0$ (D) $y + z + 1 = 0$

Ans : (D)

Hint : $\begin{vmatrix} x-1 & y-2 & z+3 \\ 2-1 & -2-2 & 1+3 \\ 1 & 0 & 0 \end{vmatrix} = 0 \quad \Rightarrow y+z+1=0$

12. Three lines are drawn from the origin O with direction cosines proportional to (1, –1, 1), (2 –3, 0) and (1, 0, 3) . The three lines are

(A) not coplanar (B) coplanar

(C) perpendicular to each other (D) coincident

Ans : (B)

Hint : $\Delta = 0$ (Coplanar)

13. Consider the non-constant differentiable function f of one variable which obeys the relation $\frac{f(x)}{f(y)} = f(x-y)$. If $f'(0) = p$ and $f'(5)=q$, then $f'(-5)$ is

(A) $\frac{p^2}{q}$ (B) $\frac{q}{p}$ (C) $\frac{p}{q}$ (D) q

Ans : (A)

Hint : $f(x) = a^{kx} \Rightarrow f'(x) = ka^{kx} \ln a$

$k \ln a = p, \quad ka^{5k} \ln a = q$

$\Rightarrow a^{5k} = \frac{q}{p}$

$\therefore f'(-5) = k.a^{-5k} \ln a = \frac{p^2}{q}$

14. If $f(x) = \log_5 \log_3 x$, then $f'(e)$ is equal to

(A) $e \log_e 5$ (B) $e \log_e 3$ (C) $\frac{1}{e \log_e 5}$ (D) $\frac{1}{e \log_e 3}$

Ans : (C)

Hint : $f(x) = \log_5 \ln x + \log_5 \log_3 e$

$f'(x) = \frac{1}{x} \cdot \frac{1}{\ln 5} \cdot \frac{1}{\ln x}$

$\therefore f'(e) = \frac{1}{e \ln 5}$

15. Let $F(x) = e^x$, $G(x) = e^{-x}$ and $H(x) = G(F(x))$, where x is a real variable. Then $\frac{dH}{dx}$ at x = 0 is

(A) 1 (B) –1 (C) $-\frac{1}{e}$ (D) –e

Ans : (C)

Hint : $H(x) = e^{-e^x}$

$\therefore H'(x) = -e^{-e^x}.e^x$

$H'(0) = -\frac{1}{e}$

16. If $f''(0) = k$, $k \neq 0$, then the value of $\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is

(A) k (B) 2k (C) 3k (D) 4k

Ans : (C)

Hint : By L Hospital Rule

17. If y = $e^{m \sin^{-1} x}$, then (1–x²) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} - ky = 0$, where k is equal to

(A) m^2 (B) 2 (C) –1 (D) $-m^2$

Ans : (A)

18. The chord of the curve $y = x^2 + 2ax + b$, joining the points where $x = \alpha$ and $x = \beta$, is parallel to the tangent to the curve at abscissa x =

(A) $\frac{a+b}{2}$ (B) $\frac{2a+b}{3}$ (C) $\frac{2\alpha+\beta}{3}$ (D) $\frac{\alpha+\beta}{2}$

Ans : (D)

Hint : $2x+2a = (\beta+\alpha) + 2a$

$\Rightarrow x = \frac{\alpha+\beta}{2}$

19. Let $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19$. Then f(x) = 0 has

(A) 13 real roots (B) only one positive and only two negative real roots

(C) not more than one real root (D) has two positive and one negative real root

Ans : (C)

Hint : $f'(x) = 0$ has no real root

20. Let $f(x) = \begin{cases} \frac{x^p}{(\sin x)^q}, & \text{if } 0 < x \le \frac{\pi}{2} \\ 0, & \text{if } x = 0 \end{cases}$, (p, q, $\in \mathbb{R}$). Then Lagrange's mean value theorem is applicable to f(x) in closed interval [0, x]

(A) for all p, q (B) only when p > q (C) only when p < q (D) for no value of p, q

Ans : (B)

Hint : $\lim_{x\to 0^+} f(x) = 0$

$\Rightarrow p > q$

21. $\lim_{x\to 0} (\sin x)^{2\tan x}$

(A) is 2 (B) is 1 (C) is 0 (D) does not exist

Ans : Either B Or D

Hint : $\lim_{n\to 0^-} (\sin x)^{2\tan x}$ $\rightarrow$ Not in the domain hence does not exist, But if approached like

$$\lim_{n\to 0^-} (\sin^2 x)^{\tan x} = \lim_{n\to 0^+} (\sin^2 x)^{\tan x} = 1$$

22. $\int \cos(\log x)\,dx = F(x) + c$, where c is an arbitrary constant. Here F(x) =

(A) $x[\cos(\log x) + \sin(\log x)]$ (B) $x[\cos(\log x) - \sin(\log x)]$

(C) $\frac{x}{2}[\cos(\log x) + \sin(\log x)]$ (D) $\frac{x}{2}[\cos(\log x) - \sin(\log x)]$

Ans : (C)

Hint : $\int \cos(\log x)dx = F(x)+c$, Let log x = t, $I = \int e^t \cos t\, dt = e^t \cos t + e^t \sin t - I$,

$$\therefore I = \frac{e^t \cos t + e^t \sin t}{2} = \frac{x}{2}\left[\cos(\log x) + \sin(\log x)\right]$$

23. $\int \frac{x^2-1}{x^4+3x^2+1}dx(x>0)$ is

(A) $\tan^{-1}\left(x+\frac{1}{x}\right)+c$ (B) $\tan^{-1}\left(x-\frac{1}{x}\right)+c$ (C) $\log_e\left(\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right)+c$ (D) $\log_e\left(\frac{x-\frac{1}{x}-1}{x-\frac{1}{x}+1}\right)+c$

Ans : (A)

Hint : dividing by x^2, $\int \frac{1-1/x^2}{x^2+1/x^2+3}dx$, Let $x+\frac{1}{x}=t$, $\int \frac{dt}{t^2+1} = \tan^{-1}\left(x+\frac{1}{x}\right)+c$

24. Let $I = \int_{10}^{19} \frac{\sin x}{1+x^8}dx$. Then

(A) $|I| < 10^{-9}$ (B) $|I| < 10^{-7}$ (C) $|I| < 10^{-5}$ (D) $|I| > 10^{-7}$

Ans : (B)

Hint : $\left|\int_{10}^{19} \frac{\sin x}{1+x^8}dx\right| \le \int_{10}^{19}\left|\frac{\sin x}{1+x^8}\right|dx \le \int_{10}^{19}\frac{1}{|1+x^8|}dx$ (as $|\sin x| \le 1$) $< \int_{10}^{19} 10^{-8}dx$ $\left(\text{as } 1+x^8 > 10^8 \text{ for } 10 \le x \le 19\right)$ $=9\times10^{-8}<10^{-7}$

25. Let $I_1 = \int_0^n [x]dx$ and $I_2 = \int_0^n \{x\}dx$, where [x] and {x} are integral and fractional parts of x and $n \in \mathbb{N} - \{1\}$. Then I_1/I_2 is equal to

(A) $\frac{1}{n-1}$ (B) $\frac{1}{n}$ (C) n (D) n –1

Ans : (D)

Hint : $I_1 = \int_0^n [x]dx = \int_0^1 0\, dx + \int_1^2 1dx + \int_2^3 2dx + ... + \int_{n-1}^n (n-1)dx$, = 0+1+2+3+.....+(n-1)= $\frac{n(n-1)}{2}$,

$I_2 = \int_0^n \{x\}dx = \int_0^n x\, dx - I_1 = \frac{n^2}{2} - \frac{n(n-1)}{2} = \frac{n}{2}$, $\therefore I_1/I_2$=n–1

26. The value of $\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+....+\frac{1}{2n}\right]$ is

(A) $\frac{n\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{4n}$ (D) $\frac{\pi}{2n}$

Ans : (B)

Hint : $\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+....+\frac{1}{2n}\right] = \lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+...+\frac{n}{n^2+n^2}\right] = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+\left(\frac{r}{n}\right)^2} = \int_0^1 \frac{1}{1+x^2}dx = \pi/4$

27. The value of the integral $\int_0^1 e^{x^2}dx$

(A) is less than 1
(B) is greater than 1
(C) is less than or equal to 1
(D) lies in the closed interval [1,e]

Ans : (B)

Hint : $\int_0^1 e^{x^2}dx > 1$

28. $\int_0^{100} e^{x-[x]}dx =$

(A) $\frac{e^{100}-1}{100}$
(B) $\frac{e^{100}-1}{e-1}$
(C) 100(e–1)
(D) $\frac{e-1}{100}$

Ans : (C)

Hint : $\int_0^{100} e^{x-[x]}dx$

$= \int_0^1 e^x dx + \int_1^2 e^{x-1}dx + \int_2^3 e^{x-2}dx + ... + \int_{99}^{100} e^{x-99}dx$

$= \int_0^1 e^x dx + \int_0^1 e^x dx + \int_0^1 e^x dx + ... + \int_0^1 e^x dx$

= 100 × (e–1)

29. Solution of $(x+y)^2\frac{dy}{dx} = a^2$ ('a' being a constant) is

(A) $\frac{(x+y)}{a} = \tan\frac{y+c}{a}$, c is an arbitrary constant
(B) xy = a tan cx, c is an arbitrary constant
(C) $\frac{x}{a} = \tan\frac{y}{c}$, c is an arbitrary constant
(D) xy = tan(x+c), c is an arbitrary constant

Ans : (A)

Hint : $(x+y)^2\frac{dy}{dx} = a^2$

[Put x + y = z $\Rightarrow 1+\frac{dy}{dx} = \frac{dz}{dx}$]

$\Rightarrow z^2\left(\frac{dz}{dx}-1\right) = a^2 \Rightarrow z^2 + a^2 = z^2\frac{dz}{dx}$

$\Rightarrow \int \frac{z^2}{z^2+a^2}dz = \int dx \Rightarrow z - a\tan^{-1}\frac{z}{a} = x + C$

$\Rightarrow \frac{x+y}{a} = \tan\frac{y+c}{a}$, c is arbitrary constant

30. The integrating factor of the first order differential equation

$$x^2\left(x^2-1\right)\frac{dy}{dx}+x\left(x^2+1\right)y=x^2-1 \text{ is}$$

(A) e^x (B) $x-\frac{1}{x}$ (C) $x+\frac{1}{x}$ (D) $\frac{1}{x^2}$

Ans : (B)

Hint : $x^2\left(x^2-1\right)\frac{dy}{dx}+x\left(x^2+1\right)y=x^2-1$, I.F = $e^{\int\frac{x\left(x^2+1\right)}{x^2\left(x^2-1\right)}dx}$,

$$e^{\int\frac{x^2-1}{x\left(x^2-1\right)}+\frac{2}{x(x+1)(x-1)}dx}=e^{\int\frac{1}{x}+\frac{-2}{x}+\frac{1}{x+1}+\frac{1}{x-1}dx}, =e^{\ln\left(\frac{x^2-1}{x}\right)}=x-1/x$$

31. In a G.P. series consisting of positive terms, each term is equal to the sum of next two terms. Then the common ratio of this G.P. series is

(A) $\sqrt{5}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{5}}{2}$ (D) $\frac{\sqrt{5}+1}{2}$

Ans : (B)

Hint : $t_r = t_{r+1} + t_{r+2}$ $ar^{n-1} = ar^n + ar^{n+1}$ $\Rightarrow$ $1 = r + r^2$ $\Rightarrow$ $r = \frac{\sqrt{5}-1}{2}$

32. If $(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$, then y equals

(A) 125 (B) 25 (C) 5/3 (D) 243

Ans : (A)

Hint : $\frac{\log x\cdot\log 3x\cdot\log y}{\log 5\cdot\log x\cdot\log 3x}=3$ $\Rightarrow \log y = 3\log 5$ $\Rightarrow y = 5^3 = 125$

33. The expression $\frac{(1+i)^n}{(1-i)^{n-2}}$ equals

(A) $-i^{n+1}$ (B) i^{n+1} (C) $-2i^{n+1}$ (D) 1

Ans : (C)

Hint : $(1-i)=\frac{2}{(1+i)}$

$$\frac{(1+i)^n}{(1-i)^{n-2}}=\frac{(1+i)^n}{2^{n-2}}\cdot(1+i)^{n-2}=\frac{(1+i)^{2(n-1)}}{2^{n-2}}=\frac{(2i)^{n-1}}{2^{n-2}}=2i^{n-1}=-2i^{n+1}$$

34. Let z = x + iy, where x and y are real. The points (x, y) in the X-Y plane for which $\frac{z+i}{z-i}$ is purely imaginary lie on

(A) a straight line (B) an ellipse (C) a hyperbola (D) a circle

Ans : (D)

Hint : Let z = x + iy

$$\therefore \frac{z+i}{(z-i)} = \frac{(x+i(y+1))\,(x+i(y-1))}{(x-i(1-y))\,(x+i(1-y))}$$

$$\text{Re}\left(\frac{z+i}{z-i}\right)=0 \quad \Rightarrow \quad \frac{x^2+(y^2-1)}{x^2+(1-y)^2}=0 \quad \Rightarrow \quad x^2+y^2=1$$

35. If p, q are odd integers, then the roots of the equation $2px^2 + (2p + q)x + q = 0$ are

(A) rational (B) irrational (C) non-real (D) equal

Ans : (A)

Hint : $D = (2p + q)^2 - 8pq = (2p - q)^2 \rightarrow$ always a perfect square

36. Out of 7 consonants and 4 vowels, words are formed each having 3 consonants and 2 vowels. The number of such words that can be formed is

(A) 210 (B) 25200 (C) 2520 (D) 302400

Ans : (B)

Hint : ${}^7C_3 \times {}^4C_2 \times 5! = 25200$

37. The number of all numbers having 5 digits, with distinct digits is

(A) 99999 (B) $9 \times {}^9P_4$ (C) ${}^{10}P_5$ (D) ${}^9P_4$

Ans : (B)

Hint : $9 \times {}^9P_4$

38. The greatest integer which divides (p + 1)(p + 2)(p + 3)......(p + q) for all $p \in \mathbb{N}$ and fixed $q \in \mathbb{N}$ is

(A) p! (B) q! (C) p (D) q

Ans : (B)

Hint : This is product of 'q' consecutive natural numbers, so it will always be divisible by q!

39. Let $((1 + x) + x^2)^9 = a_0 + a_1x + a_2x^2 + ... + a_{18}x^{18}$. Then

(A) $a_0 + a_2 + ... + a_{18} = a_1 + a_3 + ... + a_{17}$

(B) $a_0 + a_2 + ... + a_{18}$ is even

(C) $a_0 + a_2 + ... + a_{18}$ is divisible by 9

(D) $a_0 + a_2 + ... + a_{18}$ is divisible by 3 but not by 9

Ans : (B)

Hint : $a_0 + a_2 + a_4 + ... + a_{18} = \dfrac{3^9+1}{2} \rightarrow$ even

40. The linear system of equations $\left.\begin{matrix} 8x-3y-5z=0 \\ 5x-8y+3z=0 \\ 3x+5y-8z=0 \end{matrix}\right\}$ has

(A) only 'zero solution'

(B) only finite number of non-zero solutions

(C) no non-zero solution

(D) infinitely many non-zero solutions

Ans : (D)

Hint : $D = \begin{vmatrix} 8 & -3 & -5 \\ 5 & -8 & 3 \\ 3 & 5 & -8 \end{vmatrix} = 0$

$D_1 = D_2 = D_3 = 0$ infinite solutions

41. Let P be the set of all non-singular matrices of order 3 over $\mathbb{R}$ and Q be the set of all orthogonal matrices of order 3 over $\mathbb{R}$. Then,

(A) P is proper subset of Q
(B) Q is proper subset of P
(C) Neither P is proper subset of Q nor Q is proper subset of P
(D) $P \cap Q = \phi$, the void set

Ans : (B)

Hint : Q is the proper subset of P

42. Let $A = \begin{pmatrix} x+2 & 3x \\ 3 & x+2 \end{pmatrix}, B = \begin{pmatrix} x & 0 \\ 5 & x+2 \end{pmatrix}$. Then all solutions of the equation det (AB) = 0 is

(A) 1, –1, 0, 2 (B) 1, 4, 0, –2 (C) 1, –1, 4, 3 (D) –1, 4, 0, 3

Ans : (B)

Hint : $\det|AB| = \begin{vmatrix} x^2+17x & 3x^2+6x \\ 8x+10 & (x+2)^2 \end{vmatrix} = 0$

$\Rightarrow$ x (x + 2) (x – 4) (x – 1) = 0

$\Rightarrow$ x = 0, –2, 1, 4

43. The value of det A, where $A = \begin{pmatrix} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{pmatrix}$ lies

(A) in the closed interval [1, 2] (B) in the closed interval [0, 1]
(C) in the open interval (0, 1) (D) in the open interval (1, 2)

Ans : (A)

Hint : det (A) = $(1 + \cos^2\theta)$

$\Rightarrow |A| \in [1, 2]$

44. Let f : $\mathbb{R} \to \mathbb{R}$ be such that f is injective and f(x) f(y) = f(x + y) for $\forall x, y \in \mathbb{R}$. If f(x), f(y), f(z) are in G.P, then x, y, z, are in

(A) A.P always (B) G.P always
(C) A.P depending on the value of x, y, z (D) G.P depending on the value of x, y, z

Ans : (A)

Hint : $f(x) = a^x$

$a^x, a^y, a^z \to$ G.P

x, y, z $\to$ A.P

45. On the set $\mathbb{R}$ of real numbers we define xPy if and only if $xy \geq 0$. Then the relation P is

(A) reflexive but not symmetric (B) symmetric but not reflexive
(C) transitive but not reflexive (D) reflexive and symmetric but not transitive

Ans : (D)

Hint : (–1, 0), (0, 2) satisfies the relation $xy \geq 0$ but (–1, 2) doesn't satisfy relation $xy \geq 0$.

46. On $\mathbb{R}$, the relation ρ be defined by '$x\rho y$ holds if and only if $x - y$ is zero or irrational'. Then
(A) ρ is reflexive and transitive but not symmetric.
(B) ρ is reflexive and symmetric but not transitive.
(C) ρ is symmetric and transitive but not reflexive.
(D) ρ is equivalence relation
Ans : (B)

47. Mean of n observations $x_1, x_2, ..., x_n$ is $\bar{x}$. If an observation x_q is replaced by x'_q then the new mean is
(A) $\bar{x} - x_q + x'_q$ (B) $\frac{(n-1)\bar{x} + x'_q}{n}$ (C) $\frac{(n-1)\bar{x} - x'_q}{n}$ (D) $\frac{n\bar{x} - x_q + x'_q}{n}$
Ans : (D)

Hint : New Mean = $\frac{\sum_{i=1}^{n} x_i - x_q + x'_q}{n} = \frac{n\bar{x} - x_q + x'_q}{n}$

48. The probability that a non leap year selected at random will have 53 Sundays is
(A) 0 (B) 1/7 (C) 2/7 (D) 3/7
Ans : (B)

49. The equation sin x (sin x + cos x) = k has real solutions, where k is a real number. Then
(A) $0 \le k \le \frac{1+\sqrt{2}}{2}$ (B) $2-\sqrt{3} \le k \le 2+\sqrt{3}$ (C) $0 \le k \le 2-\sqrt{3}$ (D) $\frac{1-\sqrt{2}}{2} \le k \le \frac{1+\sqrt{2}}{2}$
Ans : (D)
Hint : sin 2x – cos 2x = 2k – 1

$\Rightarrow -\sqrt{2} \le 2k - 1 \le \sqrt{2}$

$\Rightarrow \frac{1-\sqrt{2}}{2} \le k \le \frac{\sqrt{2}+1}{2}$

50. The possible values of x, which satisfy the trigonometric equation $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ are
(A) $\pm\frac{1}{\sqrt{2}}$ (B) $\pm\sqrt{2}$ (C) $\pm\frac{1}{2}$ (D) ± 2
Ans : (A)

Hint : $\frac{x-1}{x-2} + \frac{x+1}{x+2} = 1 - \frac{x-1}{x-2}\cdot\frac{x+1}{x+2}$

$\Rightarrow x^2 + x - 2 + x^2 - x - 2 = x^2 - 4 - x^2 + 1$

$\Rightarrow 2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

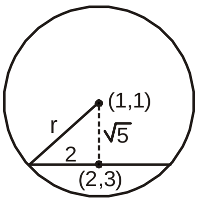
CATEGORY - II (Q51 to Q65)

Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch $-\frac{1}{2}$ marks. No answer will fetch 0 marks.

51. On set A = {1, 2, 3}, relations R and S are given by
R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}
S = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)} Then
(A) $R \cup S$ is an equivalence relation
(B) $R \cup S$ is reflexive and transitive but not symmetric
(C) $R \cup S$ is reflexive and symmetric but not transitive
(D) $R \cup S$ is symmetric and transitive but not reflexive
Ans : (C)
Hint : RUS = {(1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (3,1)}

52. If one of the diameters of the curve $x^2 + y^2 - 4x - 6y + 9 = 0$ is a chord of a circle with centre (1, 1), the radius of this circle is
(A) 3 (B) 2 (C) $\sqrt{2}$ (D) 1
Ans : (A)

Hint :



$\therefore$ r = $\sqrt{5+4} = 3$

53. Let A (–1, 0) and B (2, 0) be two points. A point M moves in the plane in such a way that $\angle MBA = 2\angle MAB$. Then the point M moves along
(A) a straight line (B) a parabola (C) an ellipse (D) a hyperbola
Ans : (D)
Hint :

54. If f(x) = $\int_{-1}^{x} |t| dt$, then for any $x \geq 0$, f(x) is equal to
(A) $\frac{1}{2}(1-x^2)$ (B) $1-x^2$ (C) $\frac{1}{2}(1+x^2)$ (D) $1+x^2$
Ans : (C)
Hint : f(x) = $\int_{-1}^{x} |t| dt$, $x \geq 0 = \frac{1}{2}(1^2 + x^2)$

55. Let for all x > 0, f(x) = $\lim_{n\to\infty} n\left(x^{\frac{1}{n}} - 1\right)$, then
(A) $f(x) + f\left(\frac{1}{x}\right) = 1$ (B) f(xy)=f(x)+f(y) (C) f(xy)=x f(y) + yf(x) (D) f(xy) = x f(x) + y f(y)
Ans : (B)

Hint : $f(x) = \lim_{n\to\infty} \frac{x^{1/n} - 1}{1/n} = \log x$

56. Let $I = \int_0^{100\pi} \sqrt{(1-\cos 2x)}\, dx$, then

(A) I = 0 (B) $I = 200\sqrt{2}$ (C) $I = \pi\sqrt{2}$ (D) I = 100

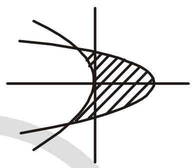
Ans : (B)

Hint : $I = \int_0^{100\pi} \sqrt{1-\cos 2x}\, dx = 100\sqrt{2} \int_0^{\pi} |\sin x|\, dx = 200\sqrt{2}$

57. The area of the figure bounded by the parabolas $x = -2y^2$ and $x = 1 - 3y^2$ is

(A) $\frac{4}{3}$ square units (B) $\frac{2}{3}$ square units (C) $\frac{3}{7}$ square units (D) $\frac{6}{7}$ square units

Ans : (A)



Hint : Curves intersect at (–2, ±1)

$$\text{Area} = 2\int_0^1 \left(1 - y^2\right) dy = 2\left(1 - \frac{1}{3}\right) = \frac{4}{3}$$

58. Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at the ends of both latus rectum. The area of the quadrilateral so formed is

(A) 27 sq. units (B) $\frac{13}{2}$ sq. units (C) $\frac{15}{4}$ sq. units (D) 45 sq. units

Ans : (A)

Hint : Equation of tangent in quadrilateral I : $\frac{2x}{9} + \frac{y}{3} = 1$

59. The value of K in order that f(x) = sin x – cos x – Kx + 5 decreases for all positive real values of x is given by

(A) K < 1 (B) $K \geq 1$ (C) $K > \sqrt{2}$ (D) $K < \sqrt{2}$

Ans : (C)

Hint : f′(x) = cosx + sinx – k (<0)

$\therefore$ k > cosx + sinx

max. (cosx + sinx) = $\sqrt{2}$

$\therefore k > \sqrt{2}$

60. For any vector $\vec{x}$, the value of $\left(\vec{x} \times \hat{i}\right)^2 + \left(\vec{x} \times \hat{j}\right)^2 + \left(\vec{x} \times \hat{k}\right)^2$ is equal to

(A) $|\vec{x}|^2$ (B) $2|\vec{x}|^2$ (C) $3|\vec{x}|^2$ (D) $4|\vec{x}|^2$

Ans : (B)

Hint : $= |\vec{x}|^2 (\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$

$= 2|\vec{x}|^2$

61. If the sum of two unit vectors is a unit vector, then the magnitude of their difference is

(A) $\sqrt{2}$ units (B) 2 units (C) $\sqrt{3}$ units (D) $\sqrt{5}$ units

Ans : (C)

Hint : $|\vec{a}-\vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}}$

$= \sqrt{3}$

62. Let α and β be the roots of $x^2 + x + 1 = 0$. If n be positive integer, then $\alpha^n + \beta^n$ is

(A) $2\cos\frac{2n\pi}{3}$ (B) $2\sin\frac{2n\pi}{3}$ (C) $2\cos\frac{n\pi}{3}$ (D) $2\sin\frac{n\pi}{3}$

Ans : (A)

Hint : $e^{i\frac{2n\pi}{3}} + e^{-i\frac{2n\pi}{3}} = 2\cos\left(\frac{2n\pi}{3}\right)$

63. For real x, the greatest value of $\frac{x^2+2x+4}{2x^2+4x+9}$ is

(A) 1 (B) –1 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans : (C)

Hint : $y = \frac{x^2+2x+4}{2x^2+4x+9}$ or, $(2y–1)(7y–3) \le 0$ or, $\frac{3}{7} \le y \le \frac{1}{2}$

64. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. Then for positive integer n, A^n is

(A) $\begin{pmatrix} 1 & n & n^2 \\ 0 & n^2 & n \\ 0 & 0 & n \end{pmatrix}$ (B) $\begin{pmatrix} 1 & n & n\left(\frac{n+1}{2}\right) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & n^2 & n \\ 0 & n & n^2 \\ 0 & 0 & n^2 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & n & 2n-1 \\ 0 & \frac{n+1}{2} & n^2 \\ 0 & 0 & \frac{n+1}{2} \end{pmatrix}$

Ans : (B)

Hint : $A = B + I \Rightarrow A^n = \begin{pmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$

65. Let a, b, c be such that $b(a+c)\neq 0$

If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$, then the value of n is

(A) any integer (B) zero (C) any even integer (D) any odd integer

Ans : (C)

Hint : $|A| = |A^T|$ or, $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - \begin{vmatrix} (-1)^{n+2}a & a+1 & a-1 \\ (-1)^{n+1}b & b+1 & b-1 \\ (-1)^{n}c & c-1 & c+1 \end{vmatrix} = 0 \Rightarrow (n+2)$ is even or n is even

CATEGORY - III (Q66 to Q75)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answer marked ÷ actual number of correct answers.

66. Let $f : \mathbb{R} \to \mathbb{R}$ be twice continuously differentiable. Let $f(0) = f(1) = f'(0) = 0$. Then

(A) $f''(x) \neq 0$ for all x (B) $f''(c) = 0$ for some $c \in \mathbb{R}$ (C) $f''(x) \neq 0$ if $x\neq 0$ (D) $f'(x) > 0$ for all x

Ans : (B)

Hint : Applying Rolle's theorem twice $f''(x) = 0$ for some $x \in [0, 1]$

67. If $f(x) = x^n$, n being a non-negative integer, then the values of n for which $f'(\alpha+\beta) = f'(\alpha) + f'(\beta)$ for all $\alpha, \beta > 0$ is

(A) 1 (B) 2 (C) 0 (D) 5

Ans : (B, C)

68. Le f be a non-constant continuous function for all $x\geq 0$. Let f satisfy the relation $f(x)\, f(a-x) = 1$ for some $a \in \mathbb{R}^+$. Then

$I = \int_0^a \frac{dx}{1+f(x)}$ is equal to

(A) a (B) $\frac{a}{4}$ (C) $\frac{a}{2}$ (D) f(a)

Ans : (C)

Hint : $I = \int_0^a \frac{dx}{1+f(x)} = \int_0^a \frac{dx}{1+f(a-x)} = \int_0^a \frac{f(x)dx}{1+f(x)} \Rightarrow 2I = \int_0^a dx \Rightarrow I = a/2$

69. If the line $ax + by + c = 0$, $ab \neq 0$, is a tangent to the curve $xy = 1-2x$, then

(A) $a > 0, b < 0$ (B) $a > 0, b > 0$ (C) $a < 0, b > 0$ (D) $a < 0, b < 0$

Ans : (B,D)

Hint : $\frac{dy}{dx} < 0$

70. Two particles move in the same straight line starting at the same moment from the same point in the same direction. The first moves with constant velocity u and the second starts from rest with constant acceleration f. Then

(A) they will be at the greatest distance at the end of time $\frac{u}{2f}$ from the start

(B) they will be at the greatest distance at the end of time $\frac{u}{f}$ from the start

(C) their greatest distance is $\frac{u^2}{2f}$

(D) their greatest distance is $\frac{u^2}{f}$

Ans : (B, C)

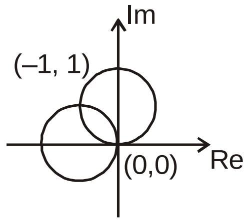
Hint : $S = ut - \frac{1}{2}ft^2$

71. The complex number z satisfying the equation $|z-i| = |z+1| = 1$ is

(A) 0 (B) $1 + i$ (C) $-1 + i$ (D) $1 - i$

Ans : (A,C)

Hint :



72. On $\mathbb{R}$, the set of real numbers, a relation ρ is defined as '$a\rho b$' if and only if $1 + ab > 0$'. Then

(A) ρ is an equivalence relation
(B) ρ is refelxive and transitive but not symmetric
(C) ρ is reflexive and symmetric but not transitive
(D) ρ is only symmetric

Ans : (C)

73. If $a, b \in \{1, 2, 3\}$ and the equation $ax^2 + bx + 1 = 0$ has real roots, then

(A) $a > b$
(B) $a \le b$
(C) number of possible ordered pairs (a, b) is 3
(D) $a < b$

Ans : (C, D)

Hint : (1, 2) (1, 3) (2, 3)

74. If the tangent to $y^2 = 4ax$ at the point $(at^2, 2at)$ where $|t|>1$ is a normal to $x^2 - y^2 = a^2$ at the point $(a \sec\theta, a \tan\theta)$, then

(A) $t = -\operatorname{cosec}\theta$
(B) $t = -\sec\theta$
(C) $t = 2\tan\theta$
(D) $t = 2\cot\theta$

Ans : (A, C)

Hint : $x - yt = -at^2$ or, $\frac{x}{a\sec\theta} + \frac{y}{a\tan\theta} = 2 \Rightarrow t = -\operatorname{cosec}\theta$ or $t = 2\tan\theta$

75. The focus of the conic $x^2 - 6x + 4y + 1 = 0$ is

(A) (2, 3) (B) (3, 2) (C) (3, 1) (D) (1, 4)

Ans : (C)

Hint : $(x-3)^2 = -4(y-2)$

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