VARDHAMAN MAHAVEER OPEN UNIVERSITY, KOTA
Rawatbhata Road, Kota (Rajasthan) - 324021

Post Graduate Degree Programme

M.A./M.Sc. Mathematics (Final)

Internal Assignments

MA/MSc MT-06 To MA/MSc MT-10

Session: 2013 - 2014

Vardhaman Mahaveer Open University, Kota
Post Graduate Degree Programme
M.A./M.Sc. Mathematics (Final)
Internal Assignments for MA/MSc MT-06 to MA/MSc MT-10

Dear Students,

The following internal assignments of various papers of M.A./M.Sc. Mathematics (Final) are being send to you:

Programme Code	Name of the Course/paper
MA/MSc MT-06	Analysis and Advanced Calculus
MA/MSc MT-07	Viscous Fluid Dynamics
MA/MSc MT-08	Numerical Analysis
MA/MSc MT-09	Integral Transforms and Integral Equations
MA/MSc MT-10	Mathematical Programming

It is must to complete the internal assignments and after completion submit the assignments to the Director of your concerned Regional Centre either through your own presence or through registered speed post. Each internal assignment is of 20 marks, the marks obtains in internal assignment will be added with the marks obtained in term end examination. It is mandatory to complete the assignments in your own hand writing. There is no revaluation system for the internal assignment except technical mistakes. After submission of the assignment you will not be given the chance to improve the same or resubmit the same so try to give the best answer in your 1st attempt. Enclose the internal assignments of each paper/course in separate files and provide the below information on the first page of each file:

Vardhaman Mahaveer Open University, Kota
Post Graduate Degree Programme
M.A./M.Sc. Mathematics (Final)

Scholar No. ☐☐☐☐☐☐☐☐☐☐☐

Name of Student .... Internal Assignment No. ....

Father's Name .... Programme Code ....

Address .... Name of the Course/Paper ....

....

Name of Study Centre .... Assignment Submission Date ....

Name of Regional Centre ....

Note :

1. Use only A4 Size Paper for your response sheets.
2. Last date of Submission : Before one month from the date of commencement of Term-End Examinations.

Internal Assigment-I
MA/MSc MT-06
Analysis and Advanced Calculus

Max. Mark : 20

Note : Attempt all the questions. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.
Attempt all the 3 questions.

1. Show that every normed space is metric space but the converse is not universally true.

OR

If T is a linear tranformation of a normed linear space N into a normed linear space N', then show that T is bounded if T maps bounded sets in N into bounded sets in N'. (7 marks)

2. Let f be a differentiable function on a non-void connected open subset V of a Banach space X over K into a Banach space Y over K such that $Df = 0$, then f is a constant function.

OR

Define regulated function. If f be a regulated function on a compact interval $[a,b]$ of R into a Banach space X. Prove that at each $t \in [a,b]$ the function $F:[a,b] \rightarrow X, F(t) = \int_a^t f$, $t \in [a,b]$ is continuous.

(7 marks)

3. Attempt any 2 parts
   (i) State and prove parseval's inequality.
   (ii) State and prove Hahn-Banach Theorem
   (iii) An operator T on a complex Hilbert space H is self adjoint iff (Tx, x) is real for all x.
   (iv) Define (i) Inner product space and (ii) Hilbert space and give an example.

(6 marks)

Internal Assigment-II
MA/MSc MT-06
Analysis and Advanced Calculus

Max. Mark : 20

Note : Attempt all the questions. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.
Attempt all the 3 questions.

1. Give an example to show that a closed and bounded subset of normed linear space need not be compect.

OR

If be a closed linear subspace of a Hilbert space H and $A^{\perp}$ be the orthogonal complement of A in H. Show that the Hilbert space H is the direct sum of the linear space A and $A^{\perp}$.

(7 marks)

2. State and prove Risez Representation Theorem.

OR

Prove that conjugate space H^{**} of H^{*} is a Hilbert space with some Inner product defined on it.

(7 marks)

3. Attempt any 2 parts
   - (i) If T be a linear transformation from a Normed linear space N into normed space N', Then T is continuous either at every point or at no point of N.
   - (ii) Show that dual of R^n is R^n.
   - (iii) Let T be a self adjoint operator then $T+T^*$ and TT^* and self adjoint.
   - (iv) If T is normal Operator on a Hilbert space H, then eigen spaces of T are pairwise orthogonal.

(6 marks)

Internal Assigment-I
MA/MSc MT-07
Viscous Fluid Dynamics

Max. Mark : 20

Note : Attempt all the questions. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three Questions :

1- Discuss Stoke's first problem.

OR

Derive Navier stoke's equation of motion for viscous fluid. (7 Marks)

2- Obtain Crocco's first integral for Pr = 1.

OR

Obtain Velocity distribution for plane couette flow and generalized couette flow. (7 Marks)

3- Attempt any two parts of the following four parts. Each part consists of three marks.

(i) Explain significance of following non-dimensional numbers.
(a) Reynolds number (b) Froude number

(ii) Write short note on boundary layer.

(iii) Explain theory of very slow motion.

(iv) Explain dynamical Similiarity. (6 Marks)

Internal Assigment-II
MA/MSc MT-07
Viscous Fluid Dynamics

Max. Mark : 20

Note : Attempt all the questions. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three Questions :

1- Discuss most general motion of a fluid element.

OR

Establish relation between stress and rate of strain components. (7 Marks)

2- Discuss the flow in tube of circular cross section.

OR

Discuss Karman flow due to a rotating disc. (7 Marks)

3- Attempt any two parts of the following four parts. Each part consists of three marks.

(i) Discuss stoke's second problem.

(ii) Explain : (a) boundary layer thickness.
(b) thermal boundary layer.

(iii) Explain Generalized law of heat conduction.

(iv) Explain circulation. (6 Marks)

Internal Assigment-I
MA/MSc MT-08
Numerical Analysis

Max. Mark : 20

Note : Attempt all the questions. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three Questions :

1- Find the root of the eqution

$x^3 - 5x^2 - 17x + 20 = 0$

by Regula-falsi method, correct upto five decimal places.

OR

Fit a straight line to the given data

x	1	2	3	4	6	8
y	2.4	3.1	3.5	4.2	5	6

(7 Marks)

2- Derive Muller's method to solve polynomial equation $f(x) = 0$

OR

Solve following system of equations by Gauss-Jordan method.

$$3x + 2y + z = 10$$
$$2x + 3y + 2z = 14$$
$$x + 2x + 3z = 14$$

(7 Marks)

3- Attempt any two parts of the following four parts. Each part consists of three marks.

(i) Perform one iteration of Jacobi method to estimate the eigenvalues of the matrix

$$\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

(ii) Explain shooting method to solve the B.V.P.

(iii) Explain finite difference method to solve the B.V.P.

(iv) What do you understand by pivoting in solving the system of linear equations.

(6 Marks)

Internal Assigment-II
MA/MSc MT-08
Numerical Analysis

Max. Mark : 20

Note : Attempt all the questions. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three Questions :

1- Find two nearly equal roots of the equation

$x^4 - 5x^3 - 12x^2 + 76x - 79 = 0$

close to $x = 2$, Using Newton-Raphson method.

OR

Derive Bairstow's method to extract quadratic factor from a polynomial equation $f(x) = 0$

(7 marks)

2- Find the root of the equation

$x^3 - x^2 - x - 1 = 0$

by Muller's method (perform two iterations)

OR

Compute y (0.5) by Milne's method where y satisfies the differential equation

$$\frac{dy}{dt} = 2e^t - y$$

and $y(0)=2, y(0.1)=2.01, y(0.2)=2.04, y(0.3)=2.09$

(7 marks)

3- Attempt any two parts of the following four parts

(i) Explain the method to fit a curve of the form $y = ae^{bx}$ to the given data.

(ii) Derive Numerov's method to solve the BVP.

(iii) Use Picard's method to solve $\frac{dy}{dt} = 1 + ty, y(2) = 0$

and hence find y at $t = 2.2$.

(iv) Explain Graeffe's root squaring method to solve the system of linear equations.

(6Marks)

Internal Assigment-I

MA/MSc MT-09

Integral Transforms and Integral Equation

Max. Mark : 20

Note : Attempt all the questions. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all the 3 questions.

1. Evaluate

$$L^{-1}\left[\frac{e^{-a\sqrt{p}}}{p}\right] \text{ by the use of complex inversion formula}$$

OR

Solve

$\frac{\partial\theta}{\partial t} = K\frac{\partial^2\theta}{\partial t^2}$ where $-\infty < x < \infty$ and $\theta = f(x)$ when $t = 0, f(x)$ being a given function of x, using fourier transform (7 marks)

2. Find the eigen valus and eigen function of the nomogeneous integral equation

$$g(x) = \lambda\int_0^{\pi}\left[\left(\cos^2 x\cos 2t + \cos 3x\cos^3 t\right)\right]g(t)dt$$

OR

Find the resovent kernel of $g(x) = 1 + \lambda\int_0^1 (1 - 3xt)g(t)dt$, find the solution of the integral equation. (7 marks)

3. Explain any 2 parts

(i) Find laplace transform of $\bar{e}^{2t}(3\cos 6t - 5\sin 6t)$

(ii) Find Hankel tranfrom of $f(x) = \begin{cases} 1 & 0<x<a\ ,\ \nu>0 \\ 0 & x>0\ ,\ \nu=0 \end{cases}$

(iii) Solve $g(x) = e^x + \lambda\int_0^1 2e^x e^t\ g(t)\ dt$

(iv) Convert the problem $y'' + \lambda y = 0, y(0) = y'(0), y(\pi) = y'(\pi)$ to an integral equation Name the integral equation also. (6 marks)

Internal Assigment-II

MA/MSc MT-09

Integral Transforms and Integral Equation

Max. Mark : 20

Note : Attempt all the questions. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all the 3 questions.

1. Find $L^{-1}\left[\frac{\cosh u\sqrt{p}}{p\cosh\sqrt{p}}\right]$ where $0<u<1$

OR

Solve $\frac{\partial V}{\partial t}=\frac{\partial^2 V}{\partial x^2}$ If $V(0,t)=0, V(x,0)=e^{-x}, x>0$ *and* $V(x,t)$ is bounded where $x>0,\ t>0$ (7 marks)

2. Show that the homogonous integral equations

$$g(x)=\lambda\int_0^1\left(t\sqrt{x}-x\sqrt{t}\right)g(t)dt$$

do not have real eigenvalues and eigan functions

OR

Solve $g(x)=(x+1)^2+\int_{-1}^{1}\left(xt+x^2t^2\right)g(t)dt$ by using the Hilbert schmidt theorem. (7 marks)

3. Attempt any 2 parts

(i) Prove that $L\left[\int_0^t\frac{1-e^{-u}}{u}du, p\right]=\frac{1}{p}\log\left(1+\frac{1}{p}\right)$

(ii) Find $f(t)$ if its cosine transform is

$$F_c(p)=\begin{cases}\frac{1}{\sqrt{2}\,\pi}\left(a-\frac{p}{2}\right), & p<2a\\ 0, & p\geq 2a\end{cases}$$

(iii) Find the Hankel transform of order Zero of $\frac{1}{x}$ and then apply the inversion formula to get the original function.

(iv) Transform $\frac{d^2y}{dx^2}+xy=1,\ y(0)=0,\ y(1)=1$ into a integral equation. (6 marks)

Internal Assigment-I
MA/MSc MT-10
Mathematical Programming

Max. Mark : 20

Note : Attempt all the questions. Question 1&2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks. Attempt all three questions:

1. (a) A positive semi definite quadratic form $f(x)=X^T AX$ is a convex function over R^n.

   (b) The sum of convex functions is convex and if at least one of the functions is strictly convex, then theorem is strictly convex.

OR

Use Revised Simplex method to solve the L.P.P.

$$\begin{aligned} \text{Max} \quad & z=2x_1-6x_2 \\ \text{s.t.} \quad & x_1-3x_2\geq 6 \\ & 2x_1+4x_2\geq 8 \\ & -x_1+3x_2\geq 6, \qquad x_1, x_2\geq 0 \end{aligned}$$

(7 marks)

2. Find the optimum integer solution to the L.P.P.

$$\begin{aligned} \text{Max} \quad & Z=3x_1+4x_2 \\ \text{s.t.} \quad & 3x_1+2x_2\leq 8 \\ & x_1+4x_2\leq 10 \\ & x_1,x_2\leq 0 \text{, and are integers.} \end{aligned}$$

OR

Solve the following L.P.P. by branch and bound technique:

$$\begin{aligned} \text{Max} \quad & Z=7x_1+9x_2 \\ \text{s.t.} \quad & -x_1+3x_2\leq 6 \\ & 7x_1+x_2\leq 35 \\ & x_2\geq 7 \end{aligned}$$

(7 marks)

3. Attempt any two parts of the following :

(i) Obtain a set of necessary condition for the non - linear programming problem:

$$\begin{aligned} \text{Maximize} \quad & Z=x_1^2+3x_2^2+5x_3^2 \\ \text{s.t.} \quad & 5x_1+2x_2+x_3=5 \\ & x_1\ x_2\ x_3\geq 0 \end{aligned}$$

(ii) Solve the following non linear programming problem using the method of Lagrangian multipliers:

$$\begin{aligned} \text{Minimize} \quad & f(X)=x_1^2+x_2^2+x_3^2 \\ \text{S.t.} \quad & 4x_1+x_2^2+2x_3=14 \\ & x_1, x_2, x_3\geq 0 \end{aligned}$$

(iii) Prove that $f(x)=\dfrac{1}{x}$ is strictly convex for $x>0$ and strictly concave for $x<0$.

(iv) Define following:

   (a) Extreme point
   (b) Convex function
   (c) Integer Programming Problem
   (d) Hyperplane

(6 marks)

Internal Assigment-II
MA/MSc MT-10
Mathematical Programming

Max. Mark : 20

Note : Attempt all the questions. Question 1&2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks. Attempt all three questions:

1. Use Kuhn-Tucker conditions to solve the following non linear programming problem:

Max. $f(x_1, x_2) = 7x_1^2 - 6x_1 + 5x_2^2$

s.t. $x_1 + 2x_2 \leq 10$

$x_1 - 3x_2 \leq 9$

$x_1, x_2 \leq 0$

OR

Solve the following quadratic programming problem by Wolfe's method:

Min. $f(x_1, x_2) = -10x_1 - 25x_2 + 10x_1^2 + x_2^2 + 4x_1 x^2$

s.t. $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 9$

$x_1, x_2 \leq 0$ (7 marks)

2. Solve the following quadratic programming problem by Beale's method:

Max. $f(x_1, x_2) = x_1 + x_2 - x_1^2 + x_1 x_2 - 2x_2^2$

s.t. $2x_1 + x_2 \leq 1$

$x_1, x_2 \geq 0$

OR

Solve the following Convex Separable programming problem:

Min. $z = x_1^2 - 2x_1 - x_2$

s.t. $2x_1^2 + 3x_2^2 \leq 6$

and $x_1, x_2 \geq 0$ (7 marks)

3. Attempt any two parts of the following:

(i) Use dynamic programming to solve the following L.P.P.

Max. $Z = 2x_1 + 5x_2$

s.t. $2x_1 + x_2 \leq 43$

$2x_2 \leq 46$

and $x_1, x_2 \geq 0$

(ii) Derive the dual of the quadratic programming problem:

Min. $f(X) = C^T X + \frac{1}{2} X^T GX$ ...(1)

Subject to $AX \geq b$ ...(2)

Where A is an m×n real matrix and g is an n×n real positive semi definite a symmetric matrix.

(iii) Write Kuhn- Tucker conditions with proof.

(iv) Lagrangen function and saddle point. (6 marks)