Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous)
Paper Code:MT-03
Differential Equations, Calculus of Variations & Special Functions
Section – C
(Long Answers Questions)

1. Solve $2x^2 \cos y \frac{d^2y}{dx^2} - 2x^2 \sin y \left(\frac{dy}{dx}\right)^2 + x \cos y \frac{dy}{dx} - \sin y = Log\, x$

A. MT-03, P. 4

2. Solve:

   (i) $t\,(1-\log y)\,\frac{d^2y}{dx^2} + (1+\log y)\left(\frac{dy}{dx}\right)^2 = 0$

   (ii) $\frac{d^5y}{dx^5} - n^2\frac{d^2y}{dx^3} = e^{ax}$

A. MT-03, P. 17, 18

3. Solve :

   (i) $2\,\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 4 = 0$

   (ii) $a\,\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}$

A. MT-03, P. 16, 20

4. Solve :

   (i) $\frac{dy}{dx} = \cos x - y \sin x + y^2$

   (ii) $\left(\frac{d^3y}{dx^3}\right)^2 + x\,\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 0$

A. MT-03, P. 11, 15

5. Solve :

   (i) $(y^2 + z^2 + x^2)\,dx - 2xy\,dy - 2xz\,dx = 0$

   (ii) $(zy + z^2)dx - xz\,dy + xy\,dx = 0$

A. MT-03, P. 30, 33

6. Solve :

   (i) $(2xz - yz)dx + (2yz - xz)dy - (x^2 - xy + y^2)dz = 0$

   (ii) $3x^2\,dx + 3y^2\,dy - (x^3 + y^3 + e^{2z})dz = 0$

A. MT-03, P. 34, 37

7. Solve :

   (i) $yz\,(y+z)dx + zx\,(x-z)dy + xy\,(x+y)dx = 0$

   (ii) $y^2z\,(x\cos x - \sin x)\,dx + x^2z\,(y\cos y - \sin y)\,dy +$
   $xy\,(y\sin x + x\sin y + xy\cos z)\,dx = 0$

A. MT-03, P. 35, 38

8. Solve $xz^3dx - zdy + 2\,ydx = 0$

A. MT-03, P.36

9. Solve :

(i) $\frac{yz}{x^2+y^2}dx - \frac{xz}{x^2+y^2}dy - \ tan^{-1}\left(\frac{y}{x}\right) ax = 0$

(ii) $(2x + y^2 + 2xz)\ dx + 2xydy + x^2\ dz = dt$

A. MT-03, P. 31, 42

10. Solve $r + (a + b)s + abt = xy$ by monge's method

A. MT-03, P. 52

11. Solve :

(i) $t - r\,sec^4y = 2\,q\tan y$

(ii) $(rt - s^2) - s\,(\sin x + \sin y) = \sin x\ siny$

A. MT-03, P. 56, 61

12. Solve :

(i) $t + s + q = 0$

(ii) $z\,(1 + q^2)r - 2pqz\,s\,(1 + p^2)t + \ z^2(rt + s^2) + 1 + p^2q^2 = 0$

A. MT-03, P. 47, 62

13. Solve :

(i) $2s - (rt - s^2) = 1$

(ii) $x^2r + 2xys + y^2t = 0$

A. MT-03, P. 59, 54

14. Solve :

(i) $q^2r - 2pqs + p^2t = 0$

(ii) $3r + 4s + t\ 6\,(rt - \ s^2) = 1$

A. MT-03 , P. 55, 58

15. Find the characteristics of :

(i) $y^2r - x^2t = 0$

(ii) $x^2r + 2x\ sy + y^2t = 0$

A. MT-03, P. 73, 74

16. Solve the following P.D.E. by the method of separation of variables.

(i) $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ given that $u(x, 0) = 6e^{-3x}$

(ii) $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$

A. MT-03, P. 75, 76

17. (a) classify the equations :

(i) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 2\frac{\partial^2 u}{\partial x\partial y} + 2\frac{\partial^2 u}{\partial y\partial z}$

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$

(b) Solve by the method of separation of variables the PDE

$4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u$ given that $u = \ 3e^{-x} - e^{-5x}$ when t = 0

A. MT-03, P. 68, 77

18. Reduce the equation

$$xyr - (x^2 - y^2)s - xyt + by - qx = 2(x^2 - y^2)$$

to canonical form and hence solve it

A. MT-03, P. 97

19. Reduce the following PDE to canonical form.

(i) $(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = n\, y^{2n-1} \frac{\partial z}{\partial y}$

(ii) $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$

A. MT-03, P.93, 94

20. Find eigen values and eigen functions for the following boundary problems

(i) $y'' + \lambda y = 0\,; y\,(a) = 0\ and\ y(b) = 0\,; 0 < a < b\,;$ b are real arbitrary constants.

(ii) $y'' - 4y' + (4 - 9\lambda)y = 0\,; y(0) = 0, y(a) = 0\,;$ where a is a positive real constant.

A. MT-03, P. 105, 108

21. Prove that :

(i) The eigen values of strom-liouville system are real.

(ii) To every eigen value of a strom-liouville system there corresponds only one linearly independent eigen function.

A. MT-03, P. 113, 115

22. Compute the eigen values and eigen functions for boundary value problem $y'' + 2y' + (1 - \lambda)y = 0\,; y(0) = 0, y(1) = 0$. Also prove that the set of eigen functions for the given problem is an orthogonal set.

A. MT-03, P. 117

23. Find the shape of the curve on which a bead is sliding from rest and accelerated by gravity will ship (without friction in least time from one point to another.

A. MT-03, P. 129

24. (a) Find the extremum of the function:

$$F[y(x)] = \int_{x_1}^{x_2} \frac{(1 + y'^2)^{1/2}}{x} dx$$

(b) Show that the curve through (1, 0) and (2, 1) which minimize :

$\int_1^2 \frac{(1+y\prime^2)^{1/2}}{x} dx$ is a circle.

A. MT-03, P. 133

25. Test for extremum of the functional.

(a) $F[y(x)] = \int_0^1 \sqrt{1 + y'^2}\ dx,\ \ y(0) = 0,\ \ y(1) = 2$

(b) $F[y(x)] = \int_0^{\pi/2} (y'^2 - y^2)\ dx,\ \ y(0) = 0,\ \ y(\pi/2) = 1$

A. MT-03, P. 128, 135

26. Test for an extremum of the functional.

(a) $F[y(x)] = \int_0^1 (x^2 y^2 + x^2 y^1)\ dx,\ \ y(0) = 0,\ \ y(1) = 1$

(b) $F[y(x)] = \int_0^1 (y'^2 + x^2)\ dx,\ \ y(0) = 0,\ \ y(1) = 2$

A. MT-03, P.127, 129

27. (a) If $y(x)$ is a curve in internal $[a, b]$ which is a twice differentiable and satisfying the condition $y(a) = y_1$ and $y(b) = y_2$ and minimizes the functional

$F[y(x)] = \int_a^b f(x, y, y')\ dx,$ where $y' = \frac{dy}{dx}$

Then the following differential equation must be satisfied

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

(c) Obtain the Euler-Lagrange equation for the extrremals functional

$$\int_{x1}^{x2} [y^2 - yy' + y'^2]\ dx$$

A. MT-03, P. 124, 135

28. Determine the curve of prescribed length 2 l which joins the points (-a, b) and (a, b) and has its centre of gravity as low as possible.

A. MY-03, P. 152

29. Find the shape assumed by a uniform rope when suspended by its end from two points at equal heights.

A. MT-03, P. 151

30. Find enternal of the functional:

(a) $I = \int_0^1 (1 + y''^2) dc,\ y(0) = 0, t'(0) = y(1) = y'(1) = 1$

(b) $I = \int_0^{\pi/2} (y''^2 - y^2 + x^2) dx,\ y(0) = y'(0) = 0 = y(\pi/2), y'(\pi/2) = -1$

A. MT-03, P. 147-148

31. (a) Find the closed convex curve of length L that encloses greatest possible area.

(b) Find the external equation for the following functional

$$I[z(x_1, x_2] = \int_D \left[\left(\frac{\partial z}{\partial x_1}\right)^2 + \left(\frac{\partial z}{\partial x_2}\right)^2\right] dx_1 dx_2$$

A. MT-03, P. 150, 148

32. Solve the Gauss hyper geometric equation.

$$x(1 - x)\frac{d^2y}{dx^2} + [\gamma - (1 + \alpha + \beta)x]\frac{dy}{dx} - \alpha\beta y = 0$$

In series in the neighborhood of the regular singular point (i) $x = 0$ and (ii) $x = 1$

A. MT-03, P. 166

33. Solve in series:

(a) $(2 - x^2)y'' + 2xy' - 2y = 0$

(b) $2x^2y'' - xy' + (1 - x^2)y = 0$

A. MT-03, P. 161, 164

34. Solve in series :

(a) $(1 - x^2)\ y'' - 2xy' + n(n + 1)y = 0$

(b) $x(1 - x)y'' + (1 - 5x)y' - 4y = 0$

A. MT-03, P. 162, 170

35. Solve in seriesL

(a) $x^2y'' + xy' + (x^2 - 1)y = 0$

(b) $x^2y'' + (x + x^2)y' + (x - 9)y = 0$

A. MT-03, P. 173,176

36. Show that if b > 0

$$2f_1(a, b, 2b; z) = \frac{2\left\{1 - \left(\frac{z}{2}\right)\right\}^{-a}}{2^{2b-1}B(b, b)}$$

$$\int_0^{\pi/2} (\sin\phi)^{2b-1} \left[(1 + \xi\, cos\phi)^{-a} + (1 - \xi\, cos\phi)^{-a}\right] d\phi$$

Where $\xi = \frac{z}{2-z}$

Deduce that $2f_1(a, b; 2b\,; z) = 2\left(1 - \frac{1}{2}z\right)^{-a} 2f_1\left[\frac{a}{2}, \frac{a}{2}, + \frac{1}{2}; b + \frac{1}{2};\ \xi^2\right]$

A. MT-03, P. 191

37. Prove that L

(a) $2f_1\left[\frac{a}{2}, \frac{a}{2}, + \frac{1}{2}; \frac{1}{2};\ z^2\right] = \frac{1}{2}\left[(1 - z)^{-a} + (1 + z)^{-a}\right]$

(b) $2f_1(-n, a + n\,; c\,; 1) = \frac{(-1)^n\,(1+a+x)n}{(c)n}$

A. MT-03, P. 190

38. Show that of $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$,

$\sin hx = n\sin x\ 2f_1\left(\frac{1}{2} + \frac{1}{2}n, \frac{1}{2} - \frac{1}{2}n; \frac{3}{2};\ sin^2 n\right)$

and $\cos n\, x = 2f_1\left(\frac{n}{2}, -\frac{n}{2}; \frac{1}{2};\ sin^2 x\right)$

A. MT-03, P. 193

39. Prove that :

(a) $B(\lambda), (-\lambda)2f_1(a, b\,; c\,, z) = \int_0^1 t^{\lambda-1}(1 - t)^{c-\lambda-1}\ (a, b;\ \wedge; zt)dt$

Where $|z| < 1, \lambda > 0, c - \lambda > 0$

(b) $2f_1(a, b; c; z) = \frac{1}{B(b,c-b)}\int_0^1 u^{b-1}(1 - u)^{c-b-1}(1 - zu)^{-a}du$

Where c>b>0 hence prove that

$2f_1(1,2; 3; z) = \log\left\{e(1 - z)^{\frac{1}{2}}\right\}^{-2/z}$

A. MT-03, P. 191, 196

40. Prove that :

(a) $2f_1\left[\frac{a}{2}, \frac{a}{2} + \frac{1}{2}; \frac{1}{2};\ z^2\right] = \frac{1}{2}\left[(1 - z)^{-a} + (1 + z)^{-a}\right]$

(b) $\lim_{\leftrightarrow -n} \frac{1}{\Gamma(c)}\ 2f_1(a, b; c; z) = \frac{(a)_{n+1}(b)_{n+1}}{(n+1)}\ 2f_1(a + n + 1, b + n + 1; n + 2\,; z)$

A. MT-03, P. 190, 195

41. Show that:

(a) $2f_1\,[-n, a+n; c; 1) = \frac{(-1)^n(1+a-c)}{(c)n}$

(b) $\int_0^{\pi/2} \frac{d\phi}{\sqrt{1-k^2 sin^2\phi}} = \frac{\pi}{2}\, 2f_1(\frac{1}{2},\frac{1}{2}; 1;\ k^2)$

A. MT-03, P. 190, 196

42. Show that :

(a) Ґ(a) Ґ(b) $2f_1\left(a, b; \frac{1}{2}; z\right) = \int_0^\infty \int_0^\infty e^{-u-v} \cos h \left\{2\sqrt{(uv)}\, z\right\} u^{a-1} v^{b-1} du dv$

Provided Re (a) > 0 and Re (b) > 0

(b) $\int_0^t x^{\frac{1}{2}} (t-x)^{-\frac{1}{2}} [1 - x^2(t-x)^2]^{-\frac{1}{2}} = \frac{1}{2}\pi t\, 2f_1\left[\frac{1}{4},\frac{3}{4}; 1;\ \frac{t^4}{16}\right]$

A. MT-03, P. 194, 198

43. If $|z| < 1$ and $\left|\frac{z}{1-z}\right| < 1$ then

(a) $2f_1\,(a, b; c; z) = (1-z)^{-a} 2f_1(a, c-b; c;\ \frac{z}{z-1})$

(b) $2f_1\,(a, b; c; z) = (1-z)^{-b} 2f_1(c-a, b; c;\ \frac{z}{z-1})$

A. MT-03, P. 202

44. Show that :

(a) $\frac{d}{dx}\,[2f_1(a, b; c\,; x] = \frac{ab}{c}\, 2f_1(a+1, b+1\,; c+1\,; x)$

(b) $\frac{d^n}{dx^n}\,[2f_1(a, b; c\,; x] = \frac{(a)_n(b)_n}{(c)_n}\, 2f_1(a+n, b+n\,; c+n\,; x)$

A. MT-03, P. 204

45. Show that by the principle of mathematical induction:

$$\frac{d^n}{dx^n}[2f_1(a, b; c; x)] = \frac{(a)_n(b)_n}{(c)_n}\, 2f_1(a+n, b+n\,; c+n\,; x)$$

A. MT-03, P. 204

46. Show that :

(a) $\frac{d}{dx}\, 1f_1\,(a; c; n) = \frac{a}{c}\, 1f_1(a+1\,;\ c+1\,; x)$

(b) $\frac{d^n}{dx^n}\,[1f_1(a; c\,; x] = \frac{(a)_n}{(c)_n}\, 1f_1(a+n, ; c+n\,; x)$

A. MT-03, P. 211

47. If $|z| < 1$ *and* $\left|\frac{z}{1-z}\right| < 1$ *then*:

(i) $2f_1(a, b\,; c\,; z) = (1-z)^2 2f_1(c-a, b\,; c; \frac{z}{1-z})$

(ii) $2f_1(a, b\,; c\,; z) = (1-z)^{c-a-b} 2f_1(c-a\,, c-b\,; c; z)$

A. MT-03, P. 202

48. If $|z| < 1$ *and* $\left|\frac{z}{1-z}\right| < 1$ *then*:

(i) $2f_1(a, b\,; c\,; z) = (1-z)^{-a} 2f_1(a, c-a,\ ; c; \frac{z}{1-z})$

(ii) $2f_1(a, b\,; c\,; z) = (1-z)^{c-a-b} 2f_1(c-a\,, c-b\,; c; z)$

A. MT-03, P. 202

49. If $|z| < 1$ *and* $\left|\frac{z}{1-z}\right| < 1$ *then*:

$2f_1(a, b\,; c\,; z) = (1-z)^{-a} 2f_1(a, c-a, b\,; c; \frac{z}{1-z})$

and deduce that:

(i) $2f_1\left(a, 1-a\,; c\,; \frac{1}{2}\right) = 2^a\, 2f_1(a\,, c+a\,; c; -1)$

(ii) $2f_1\left(a, 1-a\,; c\,; \frac{1}{2}\right) = \frac{\Gamma\left(\frac{c}{2}\right)\Gamma\left(1+\frac{c}{2}\right)}{\Gamma\left(c+\frac{a}{2}\right)\Gamma\left(1+c-\frac{a}{2}\right)}$

A. MT-03, P. 202, 203

50. Show that by the principle of mathematical induction.

$$\frac{d^n}{dx^n}\,[1f_1(a;c;x)] = \frac{(a)_n}{(c)_n} 1f_1(a+n; c+n; x)$$

A. MT-03, P.11

51. If m is a positive integer, show that:

$$2f_1(-m, a+m; c; x) = \frac{x^{1-c}(1-x)^{c-a}}{\Gamma(m+c)}\ \Gamma(c)\frac{d^m}{dx^m}\{x^{c+m-1}(1-x)^{a-c+m}\}$$

and deduce that:

$$2f_1\left(-m, a+m; \frac{a}{2}, \frac{1}{2}; \frac{1}{2} - \frac{1}{2}\mu\right) = \frac{(\mu^2-1)^{\frac{1}{2}-\frac{1}{4}a}\,\Gamma(\frac{a}{2}+\frac{1}{2})}{2^m\Gamma(\frac{1}{2}+\frac{a}{2}+m)}\ \frac{d^m}{dx^m}\{\mu^2-1\}^{m+\frac{a}{2}-\frac{1}{2}}$$

A. MT-03, P. 212

52. Solve :

$(1-x^2)y'' - 2xy' + n(n+1)y = 0$ where n is positive integer.

A. MT-03, P. 219

53. Prove that :

(i) $\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \ \ if\ m \neq n$

(ii) $\int_{-1}^{1}[P_m(x)]^2\,dx = \frac{2}{2n\mp 1} \ \ if\ m = n$

A. MT-03, P. 225

54. Prove that :

(i) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x), n\ \geq 1$

(ii) $nP_n(x) = xP_n^1(x) - P_{n-1}^1(x)$

A. MT-03, P. 226

55. Prove that:

(i) $\int_{-1}^{1} x^2 P_{n+1}P_{n-1}\,dx = \frac{2n\,(n+1)}{(2n-1)(2n+1)(2n+3)}$

(ii) $\int_{-1}^{1} xP_n\,P_{n-1}\,dx = \frac{2n}{4n^2-1}$

A. MT-03, P. 233, 234

56. Show that :

(i) $P_n(x) = \frac{1}{\pi}\int_0^{\pi}\left[x \pm \sqrt{(x^2-1)}\cos\theta\right]^n d\theta$

(ii) $P_n(x) = \frac{1}{\pi}\int_0^{\pi}\left[x \pm \sqrt{(x^2-1)}\,cos\phi\right]^{-(n+1)} d\phi$

A. MT-03, P. 232, 234

57. Show that :

(i) $Q'_{n+1} - Q'_{n-1} = (2n+1)Q_n$

(ii) $nQ'_{n+1} + (n+1)Q'_{n-1} = (2n+1)x\,Q'_n$

A. MT-03, P. 236, 237

58. Show that :

(i) $Q'_{n+1} - Q'_{n-1} = (2n+1)Q_n$

(ii) $(2n+1)(1-x^2)Q'_n = n(n+1)\,(Q_n - Q_{n+1})$

A. MT-03, P. 236, 238

59. Show that :

(i) $nQ'_{n+1}+(n+1)Q'_{n-1} = (2n+1)x\,Q'_n$

(ii) $(2n+1)x\,Q_n = (n+1)\,Q_{n+1} + nQ_{n-1}$

A. MT-03, P. 237, 238

60. Prove that $(x^2-1)\left(Q_n P'_n - P_n Q'_n\right) = C\ and$ deduce that :

(i) $\frac{Q_n}{P_n}\int_x^\infty \frac{dx}{(x^2-1)P_n^2}$

(ii) $Q_0(n) = \frac{1}{2}log\left(\frac{x+1}{x-1}\right)$

A. MT-03, P. 240

61. Show that :

(i) $x\,J'_n(x) = n\,J_n(x) - \,x\,J_{n+1}(x)$

(ii) $x\,J'_n(x) = x\,J_{n-1}(x) - \,x\,J_n(x)$

A. MT-03, P. 252, 253

62. Show that :

(i) $x\,J'_n(x) = n\,J_n(x) - \,x\,J_{n+1}(x)$

(ii) $\frac{d}{dx}[x^{-n}\,J_n(x)] = -x^{-n}\,J_{n+1}(x)$

A. MT-03, P. 252, 253

63. Show that:

(i) $x\,J'_n(x) = n\,J_{n-1}(x) - \,n\,J_n(x)$

(ii) $\frac{d}{dx}[x^n\,J_n(x)] = x^n\,J_{n-1}(x)$

A. MT-03, P. 253, 254

64. By using generating function or Bessel function show that:

(i) $\cos(x\sin\theta) = J_0 + 2J_2\cos\theta + 2J_4\cos 4\theta + \cdots ....$

(ii) $\mathrm{Sin}(x\sin\theta) = 2J_1\,\sin\theta + 2\sqrt{3}\sin\,3\theta + \cdots ....$

(iii) $\cos \mathrm{x} = J_0 - 2J_2 + 2J_4 - \cdots .....$

(iv) $\sin\,\mathrm{x} = 2J_1 - 2J_3 + 2J_5 - \cdots ....$

A. MT-03, P. 255

65. If $\lambda_i\ and\ \lambda_j$ are the roots of equation $J_n(\lambda_a) = 0$ then

$$\int_0^\infty J_n(\lambda_i, x) J_n\left(\lambda_j, x\right) dx = \begin{cases} 0 & if\ i \neq j \\ \frac{a^2}{2} J_{n+1}^2 & (\lambda, a)\ \ if\ i = j \end{cases}$$

A. MT-03, P. 260

66. Prove that :

(i) $\sqrt{\pi}\left(\frac{x}{2}\right)^{-n}\left(n+\frac{1}{2}\right)J_n\,(x) = \int_{-1}^{1}\exp(ixt)(x-t^2)^{n-\frac{1}{2}}\,dt, (n > -\frac{1}{2})$

(ii) $\int_0^a (a^2-x^2)\,J_0(kx)dx = \frac{4a}{k^3}\,J_1(ak) - \frac{2a^2}{k^2}J_0(ak)$

A. MT-03, P. 264, 266

67. Prove that:

(i) $\frac{d}{dx}\,[xJ_n(x)J_{n+1}(x)] = x[J_n^2(x) - J_{n+1}^2(x)]$

(ii) $\frac{d}{dx}\,\left[xJ_n{}^2(x)J_{n+1}^2(x)\right] = 2\left[\frac{n}{x}\,J_n^2(x) - \frac{n+1}{x}\,J_{n+1}^2(x)\right]$

A. MT-03, P. 255, 256

68. Show that :

(i) $2xH_n(x) = 2n\,H_{n-1}(x) + H_{n+1}(x)$

(ii) $H'_n(x) = 2x\,H_{n-1}(x)\ \ (n \geq 1)$

(iii) $H'_n(x) = 2x\,H_{n-1}(x) - H_{n+1}(x)$

(iv) $H''_n(x) - 2x\,H'_n(x) + 2n\,H_n(x) = 0$

A. MT-03, P. 273

69. Prove that :

(i) If $m \times n$ $\frac{d^m}{dx^m}[H_n(x)] = \frac{2^m \llcorner n}{\llcorner n-m} H_{n-m}(x)$

(ii) $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$

A. MT-03, P. 275, 276

70. Prove that :

(i) $\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x) dx = \begin{cases} 0, & if\ m \neq n \\ \sqrt{\pi}\, 2^n \llcorner n & if\ m = n \end{cases}$

(ii) $H_n(x) = 2^n \left\{ \exp\left(-\frac{1}{4}\frac{d^2}{dx^2}\right) \right\} x^n$

A. MT-03, P. 278, 279

71. Prove that :

(i) $P_n(x) = \frac{2}{\llcorner n \sqrt{\pi}} \int_0^{\infty} e^{-t^2}\, t^n\, H_n(xt) dt$

(ii) $\sum_{n=0}^{\infty} \frac{H_{n+s}(x) t^n}{\llcorner n} = \exp(2xt - t^2) Hs\,(x-t)$

A. MT-03, P. 281, 282

72. Show that :

(i) $(n+1) \llcorner n+1\,(x) = (2n+1-x) \llcorner n(x) - n \llcorner n-1\,(x)$

(ii) $x \llcorner n'(x) = n \llcorner n(x) - n \llcorner n-1(x)$

A. Mt-03, P. 288, 289

73. Establish the generating functions :

(i) $\Gamma(1+\alpha)(xt)^{-\frac{\alpha}{2}} e^t J_n(2\sqrt{xt}) = \sum_{n=0}^{\infty} \frac{1}{(1+\alpha)_n} \llcorner_n^{\alpha}\,(x) t^n$

(ii) $\frac{1}{(1-t)^c} 1F_1\left(C\,; 1+\alpha;\, -\frac{xt}{1-t}\right) = \sum_{n=0}^{\infty} \frac{(c)_n}{(1+\alpha)_n} \llcorner_n^{\alpha}(x) t^n$

A. MT-03, P. 300, 301

74. Prove that :

(i) $\int_0^{\infty} e^{-st} \llcorner_n(t) dt = \frac{1}{s}\left(1 - \frac{1}{s}\right)^n$

(ii) $\frac{d}{dx} \llcorner_n^k(x) = -\llcorner_{n-1}^{k+1}\,(x)$

A. MT-03, P. 292, 297

75. Show that:

(i) $\llcorner_{n+1}^{j}(x) + \llcorner_n^{k+1}(x) = \llcorner_n^k(x)$

(ii) $(n+1)\llcorner_{n+1}^k = (2n+k+1-x)\llcorner_n^k\,(x) - (n+k)\llcorner_{n-1}^k(x)$

A. MT-03, P. 295

76. Prove that :

(i) $\int_0^{\infty} e^{-x} \llcorner_m(x) \llcorner_n(x) dx = \begin{cases} 0 & if\ m \neq n \\ 1 & if\ m = n \end{cases}$

(ii) $\frac{1}{(1-t)^{k+1}}\, exp\left\{-\frac{xt}{1-t}\right\} = \sum_{n=0}^{\infty} L_n^k(x) t^n$

A. MT-03, P. 290, 294