

# ASSAM SCIENCE AND TECHNOLOGY UNIVERSITY

Guwahati

**Course Structure and Syllabus** 

(From Academic Session 2018-19 onwards)

M.Sc. Mathematics (CBCS)

4<sup>th</sup> Semester



## ASSAM SCIENCE AND TECHNOLOGY UNIVERSITY

## Course Structure (From Academic Session 2018-19 onwards)

## M.Sc. Mathematics (CBCS)

4<sup>th</sup> Semester

Sl No.	Sub-Code	Subject	Hours per Week		Credit	Ma	Marks	
			L	Т	Р	С	CE	ESE
Theo	Theory							
Core			<b>r</b>			1		
1	MMA182401	Measure Theory	4	2	0	6	30	70
2	MMA182402	Non-linear Dynamical Systems and Chaos	5	2	0	7	30	70
3	MMA182403	Discrete Structure and Graph Theory	4	2	0	6	30	70
Opti	onal (Any One Pap	er)						
1	MMA18240E21	Magnetohydrodynamics	4	2	0	6	30	70
2	MMA18240E22	Mathematical Modeling	4	2	0	6	30	70
3	MMA18240E23	Computational Fluid Dynamics	4	2	0	6	30	70
4	MMA18240E24	Mechanics of Solids-II	4	2	0	6	30	70
5	MMA18240E25	Operator Theory	4	2	0	6	30	70
6	MMA18240E26	Algebraic Topology	4	2	0	6	30	70
7	MMA18240E27	Algebraic Graph Theory	4	2	0	6	30	70
8	MMA18240E28	Theory of Modules	4	2	0	6	30	70
9	MMA18240E29	Algebraic Number Theory	4	2	0	6	30	70
10	MMA18240E210	Fields and Galois Theory	4	2	0	6	30	70
	Total 17 8 0 25 120 280					280		
Tota	Total Contact Hours per week : 25							
Tota	Total Credit : 25							

#### **Detailed Syllabus:**

Course Code	Course Title	Hours per week L-T-P	Credit C
MMA182401	Measure Theory	4-2-0	6

#### Unit 1: Measurable Sets

Outer measure, Lebesgue measure, measurable sets and their properties, Borel sets, Characterization of measurable sets, non-measurable sets.

#### **Unit 2: Measurable Functions**

Properties, Step functions, Characteristic functions, Simple functions, Continuous functions, Set of measure zero, Borel measurable function, Realization of non-negative measurable functions in terms of simple functions, Convergence in measure.

#### **Unit 3: Lebesgue Integrals**

Riemann integrals, Lebesgue integration of a simple function, Bounded convergence theorem, Fatou's lemma, Monotonic Convergence Theorem, integrable functions, General Lebesgue Integral, Dominated convergence theorem.

#### **Unit 4: Differentiation and Indefinite integrals**

Dini Derivatives, functions of bounded variation, Jordan decomposition Theorem, Indefinite integrals, Signed measures and their derivatives, Hahn decomposition, Radon Nykodym theorem.

#### Unit 5: L<sup>p</sup> –Space

The  $L^p$  space, Holder, Minkowski''s inequalities, summable sequence, essential supremum, Completeness of  $L^p$  space, Riesz-Fischer theorem, Bounded linear functional on  $L^p$  spaces, Riesz representation theorem.

#### **Text Books:**

1.H.L. Royden, *Real Analysis*, Mc-Millan 2.G.D. Berra, *Measure Theory and Integration*, Wiley Eastern LTD

#### **Reference Books:**

1. W. Rudin, Principles of Mathematical Analysis (Ed-3), McGraw Hill

Course Code	Course Title	Hours per week L-T-P	Credit C
MMA182402	Non-linear Dynamical Systems and Chaos	5-2-0	7

To build a strong foundation for Nonlinear Dynamics through Difference Equations and Differential Equations.

#### **Unit-1: One Dimensional Flows and Bifurcations**

Introduction, Fixed points and Stability, Population Growth, Linear Stability Analysis, Existence and Uniqueness, Impossibility of oscillations, Potentials, Saddle-node bifurcation, Transcritical bifurcation, Pitchfork bifurcation, Imperfect bifurcations, Flow on the circle.

## **Unit-2: Two Dimensional Flows and Bifurcations**

Linear Systems: Definition, examples and classification of linear systems, Phase planes: Introduction, phase portraits, conservative systems, Reversible systems, Index theory,

Limit cycles: Introduction and examples, ruling out closed orbits, Liapunov Functions, Poincare-Bendixson, theorem, Lienard Systems, Relaxation Oscillators, Weakly non-linear oscillators, Saddle-node bifurcation, Transcritical bifurcation, Pitchfork bifurcation, Hopf bifurcation.

## Unit 3: Chaos

Lorenz Equations: Introduction, Simple properties of the Lorenz equation, Definitions of chaos, attractors and strange attractors,

One dimensional maps: Introduction, Fixed points and Cobwebs, Numeric and analysis of Logistic map, Renormalization,

Fractals: Countable and uncountable sets, Cantor set and its fractal property, Dimensions of self similar fractals, Box Dimension, The von Koch curve, Strange attractors, The Baker's map B.

## **Text Books:**

- 1. Nonlinear Dynamics and Chaos by Steven H. Strogatz Westview Press, ISBN 13 978-0-7382-0453-6
- 2. Understanding Nonlinear Dynamics, Author Daniel Kaplan and Leon Glass, Springer, New York.

#### **Reference Books:**

- 1. Nonlinear Dynamics and Chaos by Thompson JMT and Stewart H B John Wiley and Sons, Chichester.
- 2. An Introduction to Chaotic Dynamical systems by Robert L Devancy Addison-Wesley Publishing Company Inc. 1989
- 3. Nonlinear Systems by P.G.Drazin Cambridge University Press.

#### Website and Elearning Source:

http://mathforum.org,http://ocw.mit.edu/ocwweb/Mathematics,http://www.opensource.org, www.algebra.com

<b>Course Code</b>	Course Title	Hours per week	Credit
		L-T-P	С
MMA182403	Discrete Structure and Graph Theory	4-2-0	6

Students will learn few interesting topics of Discrete Structure as well as certain fascinating applications of different types of Graphs.

#### **Discrete Structure**

#### **Unit – 1: Grammars and Languages**

Definitions and Examples, Context- free grammar, Regular grammar, Operations on Languages, Regular Grammar, Finite State Automata: State diagram of an Automata.

## **Graph Theory**

#### **Unit - 2: Graphs and Trees**

Graph, Basic definitions, Isomorphism of graphs, Subgraphs, Walks, Paths, Circuits, Connected graphs, Disconnected graphs, Trees, Some properties of trees, Distance and centers in a tree, Rooted and binary trees, On counting trees, Spanning trees, Cut-sets, Some properties of a cut-set, Connectivity and Separability, Blocks.

#### **Unit – 3: Operations On Graphs**

Planar and non-planar graphs, Kuratowski"s two graphs, Different representations of a planar graph, Matrix representation of graphs, Incidence matrix, Adjacency matrix, Graph matchings, Graph coverings.

#### **Unit - 4: Directed Graphs and Enumeration of Graphs**

Definition of Directed graphs (digraph), Some types of digraphs, Digraphs and binary relations, Directed paths and connectedness, Acyclic digraphs and decyclization, Enumeration of graphs, Types of enumeration, Counting labeled trees, Counting unlabelled trees.

#### **Unit - 5: Graph Algorithms**

Algorithms, Shortest-path algorithms, Transitive closure of a digraph, Activity network, Topological sorting, Critical path, Graphs in computer programming (basic concepts).

#### **Text Books**:

- 1. Discrete Mathematical Structures with Applications to Computer Science, by J. P. Tremblay, R. Manohar, Tata McGraw Hill, 1997
- 2. Graph theory with applications to engineering and computer science by Narsigh Deo, Prentice- Hall of India Private Limited, New Delhi.
- 3. Graph Theory, by R. Haray, Narosa Publishing House, New Delhi, 2001

## **Reference Books**

- 1. Graph Theory by F. Harary, Addison Wesley, 1969.
- 2. G. Chartrand, Introductory Graph Theory, Dover Publications, 1984
- 3. J. L. Gross, J. Yellen, Handbook of Graph Theory, CRC Press, 2004

## Website and Elearning Source:

http://mathforum.org, http://ocw.mit.edu/ocwweb/Mathematics.

Course Code	Course Title	Hours per week L-T-P	Credit C
MMA18240E21	Magnetohydrodynamics	4-2-0	6

The learner will learn about the electrical properties of Fluid and the effects of magnetic fields on them.

## **Unit 1: MHD Approximations**

The electrical properties of Fluid, electric and magnetic field, Lorentz force, action at a distance, the low frequency approximations, energetic aspects of MHD, magnetic energy.

## Unit 2: The Kinematic aspects of MHD

The magnetic induction equation, the anlogy with vorticity, diffusion and convection of magnetic field, Magnetic reynold number, the dynamo problem,

Alfven"s theorems, Cowling problem, the two dimensional kinematic problem with flow in the direction of no variation, the two dimensional kinematic problem with field in the direction of no variation, the two dimensional kinematic problem with current in the direction of no variation.

#### Unit 3: The magnetic force and its effects

The magnetic force and the inertia force, magnetic stress, principal directions and stress, Magnetohydrostatic, The linear pinch confinement scheme, the force free fields, the magnetic field in moving fluid, invalidation of kelvin"s theorem on vorticty, the case of irrotational force per unit mass.

#### **Unit 4: Boundary Conditions**

Boundary conditions for magnetic field, boundary condition for current, boundary conditions for electric field, boundary condition on velocity.

#### Unit 5: Linear magnetohydrodynamics : Linearised MHD equations for

i) 1-D case: The steady Hartmaan Flow problems, Poiseuille type flow, Coutte type of Flow, Linear Alfven waves, MHD Rayleigh problem ii) 2-D case: Steady laminar flow in a pipe under uniform transverse field.

#### **Text Books:**

1. A text book of Magnetohydrodynamics, J.A. Schercliff, Pergamon Press, New York (1965).

## **Reference Books:**

1. Magnetohydrdynamics by T. G. Cowling, Interscience Publishers, 1957.

#### Website and Elearning Source:

http://mathforum.org, http://ocw.mit.edu/ocwweb/Mathematics, http://www.opensource.org

Course Code	Course Title	Hours per week	Credit
		L-T-P	С
MMA18240E22	Mathematical Modeling	4-2-0	6

Knowledge of Mathematical formulation of real life problems and techniques of solving them is expected.

## **Unit 1: Introduction**

The Technique on Mathematical Modelling, Mathematical Modelling Through Calculus, Mathematical Modelling Through Ordinary Differential Equations of first order, Linear Growth and Decay Models, Non-Linear Growth and Decay Models, Compartment Model, Mathematical Modelling in Dynamics through Ordinary Differential Equations of first order.

## **Unit 2: Application of Mathematical Modelling**:

Mathematical Modelling in Population Dynamics, Mathematical Modelling of Epidemics through systems of Ordinary Differential Equations of first order, Mathematical Modelling in Economics based on systems of Ordinary Differential Equations of first order, Mathematical Models in Medicine, Arms Race Battles and International Trade in terms of Ordinary Differential Equations.

#### **Unit 3: Modeling Through Difference Equations**

Modelling through Difference Equations, Some Simple Models, Mathematical modelling through Difference Equations in Economics, Finance, Population Dynamics and Genetics.

## **Unit 4: Modelling Through Partial Differential Equations**

Partial Differential Equation Model for Birth-Death-Immigration-Emigration Process, Partial Differential Equation Model for a Stochastic-Epidemic Process, Model for Traffic on a Highway.

#### **Text Books**:

1. Mathematical Modelling by J.N.KAPUR, Wiley Eastern Ltd, New Delhi,

#### **Reference Books**

1. An Introduction to Mathematical Modelling by EDWARD A. BENDER, John Wiley and sons, New York.

#### Website and Elearning Source:

http://mathforum.org, http://ocw.mit.edu/ocwweb/Mathematics, http://www.opensource.org,

<b>Course Code</b>	Course Title	Hours per week	Credit
		L-T-P	С
MMA18240E23	<b>Computational Fluid Dynamics</b>	4-2-0	6

The learner is expected to have both theoretical and hands on expertise in solving Fluid Dynamics problems using the CFD techniques and tools.

#### Section A: Unit – 1

Computational Fluid Dynamics, Governing Equations of Fluid Dynamics, Boundary Conditions, Forms of Governing Equations suitable for CFD, Classification of partial Differential Equations.

## **Unit – 2: Basic aspects of Discretization**

Finite difference, Difference and Transformations Equations, Explicit and Implicit Approaches, Errors and Stability, General Transformation Equations, Stretched grid, Boundary-Fitted Co-ordinate Systems.

#### **Unit – 3: CFD Techniques**

The Lax-Wendroff and MacCormack"s Techniques, Relaxation, Central Difference Equations for Navier-Stokes Equations.

#### Section B: Practical

Numerical Formulation using Crank-Nicholson Technique for Couette flow and two-dimensional problems. Program development and execution.

#### **Text Books**:

- 1. John D Anderson, Jr.: Computational Fluid Dynamics, Mc-Graw Hill
- 2. John C. Tannehill, Dale A. Anderson and Richard H. Pletchor: Computational Fluid Dynamics and Heat Transfer; Taylors and Francis.
- 3. T.J. Chung: Computational Fluid Dynamics, Cambridge Univ. Press
- 4. Tapan K. Sengupta: Computational Fluid Dynamics, University Press

#### Website and Elearning Source:

http://mathform.org.http://ocw.mit.edu/ocwweb/Mathematics, http://www.opensource.org.

Course Code	Course Title	Hours per week L-T-P	Credit C
MMA18240E24	Mechanics of Solids-II	4-2-0	6

The learner will be able to learn the mechanics of both elastic and plastic bodies.

## Unit-1: General Theorems of the theory of Elasticity

Betti's Reciprocal theorem, principle of minimum potential energy,

Castigliano's principle, Rayleigh - Ritz method, Reissner's variational principle, equilibrium equations and boundary conditions for a geometrically non-linear body.

#### Unit-2: The propagation of waves in elastic solid media

Waves of dilatation and distortion in isotropic elastic media, plane waves, Rayleigh surface waves, Love waves.

#### **Unit-3: Foundations of the theory of Plasticity**

Basic Concepts of Plasticity, the criterion of yielding, Tresca and Von Mises criteria, strain hardening, Levy-Mises and Prandtl-Reuss equations, Hencky stress strain relations.

#### **Unit-4: The solution of elasto-plastic problems**

Hohenemser"s experiment, torsion and tension of thin walled tube and cylindrical bar, bending under conditions of plain strain, bending of prismatic beam, torsion of prismatic bar, torsion of a bar of non-uniform cross-section.

#### **Unit-5: Plane Plastic Strain and theory of Slip line field**

Plastic rigid materials, Plane strain, Plain strain equations, elementary slip line theory for plan plastic strain.

#### **Text Books:**

- 1. Theory of Elasticity by Yu. A. Amenzade, MIR Publishers, Moscow.
- 2. Theory of Elasticity by S. P. Timoshenko and J. N. Goodier, McGraw Hill Education.
- 3. Mathematical Theory of Plasticity by R. Hill, Clarendon Press, Oxford.

#### **Reference Books:**

- 1. A Treatise on the Mathematical Theory of Elasticity by A. E. H. Love, Dover Publications.
- 2. Schaum's Outline of Theory and Problems of Continuum Mechanics by George E. Mase., Schaum's Outline Series, McGraw-Hill.
- 3. Fundamentals of the theory of Plasticity by L. M. Kachanov, NorthHoland publishing Company.

Course Code	Course Title	Hours per week	Credit
		L-T-P	С
MMA18240E25	<b>Operator Theory</b>	4-2-0	6

To Study the Spectral Properties of various Operators on Normed and Inner Product Spaces.

#### **Prerequisite: Functional Analysis**

#### **Unit-1: Spectral Theory of Linear Operators in Normed Spaces**

Spectral Theory in Finite Dimensional Normed Spaces and Its basic concepts, Spectral Properties of Bounded Linear Operators, Resolvent and Spectrum and Their Properties, Banach Algebra and Its properties.

#### **Unit-2: Compact Linear Operators on Normed spaces and Their Spectrum**

Compact Linear Operators on Normed Spaces and Its Properties, Spectral Properties of Compact Linear Operators, Further Spectral Properties of Compact Linear Operators.

#### **Unit-3: Spectral Theory of Bounded Self-Adjoint Linear Operators**

Adjoint Linear Operators, Self-Adjoint Linear Operators, Spectral properties of Bounded Self Adjoint Linear operators, Further Spectral Properties of Bounded Self Adjoint Linear Operators, Positive Operator, Square Root of Positive Operator, Projection operators.

#### **Unit-4: Unbounded linear operators on Hilbert Space**

Unbounded Linear Operators and Their Hilbert Adjoint operators, Symmetric and Self Adjoint linear operators, Closed Linear Operator and Closure, Multiplication and Differentiation of Operators.

#### **Text Books:**

1. Erwin Kreyszig, Introduction to Functional Analysis with Applications, John Wiley & Sons.

#### **Reference Books:**

- 1. Martin Schechter, Principles of Functional Analysis, Student Edition, Academic Press.
- 2. Balmore V. Limaye, Functional Analysis, New Age International Ltd.

## Website and Elearning Source:

http://mathforum.org,http://ocw.mit.edu/ocwweb/Mathematics,http://www.opensource.org,www. algebra.com

Course Code	Course Title	Hours per week	Credit
		L-T-P	С
MMA18240E26	Algebraic Topology	4-2-0	6

To study Homotopy Theory, Homology Theory and Covering Projections.

#### **Unit 1: Fundamental Group**

Introduction, Homotopy, Contractible Spaces and Homotopy type, Fundamental Group and its Properties, Simply Connected Spaces, The Fundamental Group of Circle.

#### **Unit 2: Finite Simplicial Complexes**

Simplicial Complexes, Polyhedra and Triangulations, Simplicial Approximation, Barycentric Subdivision, Simplicial Approximation Theorem.

#### **Unit 3: Simplicial Homology**

Introduction, Orientation of Simplicial Complexes, Simplicial Chain Complex and Homology, Some Examples.

#### **Unit 4: Covering Projections**

Introduction, properties of covering projections, applications of homotopy lifting theorem, Lifting of an arbitrary map.

#### **Text Books:**

1. Satya Deo, Algebraic Topology, A Primer, Hindustan Book Agency, New Delhi, 2003.

#### **Reference Books:**

1. B.K. Lahiri, Algebraic Topology, Narosa Publishing House, 2000

#### Website and Elearning Source:

http://mathform.org.http://ocw.mit.edu/ocwweb/Mathematics, http://www.opensource.org.

Course Code	Course Title	Hours per week	Credit
		L-T-P	С
MMA18240E27	Algebraic Graph Theory	4-2-0	6

Students will learn the algebraic extension and application of Graph Theory.

#### Unit – 1: Reviews

Basic Definitions of Graph theory and Linear Algebra, Matrix Representations of a graph: Adjacency matrix and Incidence matrix.

#### **Unit – 2: Eigenvalues of Graphs**

A Little Matrix Theory, Eigenvalues and Walks, Eigenvalues and Labeling of graphs, Lower and Upper Bounds for the Eigenvalues, S eidel matrix of a graph.

#### **Unit – 3: Graph Laplacians**

Laplacian of a graph, Laplace Eigenvalues, Eigenvalues & Vertex partition of graphs, The Max-Cut Problem, Travelling Salesman Problem, Random Walks on graphs.

#### **Unit – 4: Spectral Graph Theory**

Introduction, Angles, Star sets and Star partitions, Integral Graphs.

#### **Text Books:**

- 1. R. J. Wilson, I. W. Beineke, Topics in Algebraic Graph Theory, Cambridge University Press, 2004.
- 2. C. Godsil, G. Royle, Algebraic Graph Theory, Springer Verlag Newyork, 2001.

#### **Reference Books:**

- 1. Norman Biggs, Algebraic Graph Theory, Cambridge University Press, 1974.
- 2. U. Knauer, Algebraic Graph Theory, Hubert & Co., Germany, 2011.

#### Website and Elearning Source:

http://www.graphtheory.com/

Course Code	Course Title	Hours per week	Credit
		L-T-P	С
MMA18240E28	Theory of Modules	4-2-0	6

## UNIT-1:

Basic concepts of Modules, Direct Products and Sums, Free Modules, Bases for Free Modules

#### UNIT-2:

Injective Modules, Properties of Injectives, Divisibility, Embedding in Injectives, Injective Hulls, Noetherian Rings.

#### UNIT-3:

Tensor product of Modules, Tensor Product of Algebras, Free and Tensor Algebras, Exterior Algebras,

## UNIT-4:

Simple Modules and Primitive rings, Cyclic Modules, Density, Jacobson radical and its Characterization

#### **Text Books:**

1. John Dauns, Modules and Rings, Cambridge University Press, 1994

#### **Reference Books:**

- 1. S.Lang, Algebra, Addison-Wesley, 1994
- 2. N. Jacobson, Basic Algebra II, Hindustan Publishing Corporation, New Delhi, 2002.
- 3. M.F. Atiyah and I.G. MacDonald, Introduction to Commutative Algebra, Addison Wesley, 1969.

Course Code	Course Title	Hours per week	Credit
		L-T-P	С
MMA18240E29	Algebraic Number Theory	4-2-0	6

## **UNIT-1: Algebraic Integers**

Localisation, Integral Closure, Prime Ideals, Galois Extensions, Dedekind rings, Discrete Valuation Rings, Explicit factorisation of a prime, Projective Modules over Dedekind Rings

#### **UNIT-2: Completions**

Polynomial in complete fields, Filtrations, Unramified Extensions, Tamely ramified extensions

## UNIT-3:

Complementary Modules, The different and ramification, The discriminant, Roots of Unity, Quadratic Fields, Gauss Sums, Relations in Ideal classes.

#### **Text Books:**

1. S. Lang, Algebraic Number Theory, Addison- Wesley, 1970

#### **Reference Books:**

- 1. Richard A. Mollin, Algebraic Number Theory, CRC Press, 1999
- 2. Stewart and Tall, Algebraic Number Theory and Fermat's Last Theorem, A K Peters, 2002.

Course Code	Course Title	Hours per week	Credit
		L-T-P	С
MMA18240E210	Fields and Galois Theory	4-2-0	6

#### **Unit – 1:**

Fields and their Extensions, Splitting Fields, The Algebraic Closure of a field.

#### Unit – 2:

Separability, Automorphisms of Field Extensions.

## Unit – 3:

Fundamental Theorem of Galois Theory, Roots of Unity, Finite Fields.

## Unit – 4:

Primitive Elements, Galois Theory of Equations, Solution of Equations by Radicals.

## **Text Books:**

1. P.M. Cohn: Basic Algebra, Springer International Edition 2003

## **Reference Books:**

1. Thomas W. Hungerford, Algebra, Springer-Verlag, New York, 1974

2. Ian Stewart, Galois Theory, Chapman & Hall, 1945

3. Emil Artin, Galois Theory, University of Notice Dame Press, 1971

## Website and Elearning Source:

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