SRI CHANDRASEKHARENDRA SARASWATHI VISWA MAHAVIDHYALAYA
(Declared as Deemed-to-be University under Section 3 of the UGC Act, 1956,
Vide notification No.F.9.9/92-U-3 dated 26th May 1993 of the Govt. of India)
ENATHUR. KANCHIPURAM – 631 561

DEPARTMENT OF MECHANICAL ENGINEERING

QUESTION BANK

BOUNDARY VALUE PROBLEMS AND STATISTICS (EBU4FT051)

Staff Name	:	Mrs. R. MALATHI / Lecturer (Department of Mathematics)
Class	:	B.E. (Mechanical Engineering)
Year / Semester	:	II Year / IV Semester

UNIT- I

ONE MARK

1. Write Euler formula for the Fourier coefficients of f(x) in $(-l, l)$.
2. Find b_n in the Fourier series expansion of $f(x) = x - x^2$ in $(-\pi,\pi)$ of periodicity 2π.
3. Find a_0, $f(x) = |\sin x|$, $(-\pi, \pi)$
4. $\int e^{ax} \sin bx \, dx$ = ----------------
5. Find a_0, $f(x) = |\cos x|$, $(-\pi, \pi)$
6. Define DIRICHLET'S condition.
7. Find a Fourier cosine series for the function $f(x) = 1$, $0 < x < \pi$.

EIGHT MARKS

1. If $f(x) = \begin{cases} \pi x; & 0 \le x \le 1 \\ \pi(2-x); & 1 \le x \le 2 \end{cases}$ Show that the Fourier series of f (x) in the interval (0,2) is

   $$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \ldots\ldots\ldots\right]$$

2. Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$.

3. Obtain the Fourier series of the periodic function defined by $f(x) = -\pi$ if $-\pi < x < 0$.
   $= x$ if $0 < x < \pi$.

   Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots = \frac{\pi^2}{8}$.

4. Obtain the Fourier series of the periodic function defined by

   f(x) = 1 for $0 < x < 1$
   = 2 for $1 < x < 3$

5. Find a Fourier series to represent ½ $(\pi - x)$, $0 < x < 2\pi$

6. Expand $f(x) = \begin{cases} \frac{1}{4} - x, \text{if } 0 < x < \frac{1}{2}, \\ x - \frac{3}{4}, \text{if } \frac{1}{2} < x < 1, \end{cases}$ as a Fourier series of sine terms.

UNIT-II

ONE MARK

1. State Parseval's identity for Fourier transforms.
2. Prove the shifting property of Fourier transform of f(x).
3. Find a Fourier cosine and sine transform of $f(x) = 2e^{-5x}+5e^{-2x}$

II EIGHT MARKS

1. Using Parseval's identities prove that

a) $$\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$$

b) $$\int_0^\infty \frac{\sin at}{t(a^2+t^2)}\,dt = \frac{\pi}{2}\left(\frac{1-e^{-a^2}}{a^2}\right)$$

2. Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$ and hence derive Fourier sine transform of $\Phi(x) = \frac{x}{1+x^2}$.

3. Show that the Fourier transform of f(x) = $e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$

4. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$

UNIT- III

ONE MARK

1. Obtain the complete solution for $p - q = x - y.$
2. Find the complementry function of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$
3. Obtain the complete solution for p + q = sinx + siny
4. Write the complete solution of z = px +qy+pq ?
5. Eliminate the arbitrary constants a and b from $z = ax + by + a^2+b^2$
6. Find P.I of (D^2- 2DD') z = x^3 y

EIGHT MARKS

1. Solve x (y – z) p + y (z – x) q = z (x – y).
2. Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{2x+y}.$
3. Solve $z = px + qy + \sqrt{1+p^2+q^2}$
   (Find the complete integral and single integral)
4. Solve $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial y \partial x} + 4\frac{\partial^2 z}{\partial y^2}$ = sin (2 x + 3 y)
5. Solve (D^3- 4 $D^2 D'$ + 4 D D'^2) z = 6 sin (3 x + 6 y)
6. Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x + y).$

UNIT-IV

ONE MARK

1. Using D'Alembert's method, find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection f (x) = k (s i n x – s i n 2 x).
2. State any one assumption in deriving Wave equation.
3. State heat one dimension equation
4. Write down the various possible solutions of wave equation
5. Obtain the suitable solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

EIGHT MARKS

1. A tightly stretched string with fixed end points x =0 and x = l is initially in a position given by $y = y_0 \sin^3(\pi x/l)$. If it is released from rest from this position, find the displacement y(x,t).

2. Solve the following one dimensional heat flow equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

3. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time t = 0. Show that the displacement of any point at a distance x from one end at time 't' is given by
$y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$

4. **A homogenous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is**

$$u(x,0) = x \qquad 0 \le x \le 50$$
$$= 100 - x \qquad 50 \le x \le 100$$

5 solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ **subject to**

i) **u(0,t) = 0 for t ≥ 0**

ii) **u(** λ, t **) = 0 for t ≥ 0**

iii) **u(x , 0) =** **x ,** **o ≤ x ≤** $\frac{\lambda}{2}$

$\lambda - x$, $\frac{\lambda}{2}$ **≤ x ≤** λ

6. **If a string of length** l **is initially at rest in equilibrium position and each point of it is given the velocity** $\left(\frac{\partial y}{\partial t}\right)_{t=0} = \mathbf{v}_0 \sin^3 \frac{\pi}{l} x, \quad 0 < x < 1,$ **determine the transverse displacement** $y(x,t)$.

7. **A tightly stretched string with fixed end points** $x = 0$ **and** $x = l$ **is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity** $3x(l - x)$, **find the displacement.**

UNIT- V

ONE MARK

1. Give formula for rank correlation.
2. Give formula for correlation coefficient.
3. Define Scatter diagram

EIGHT MARKS

1. Fit an exponential curve for the following data:

x	0	2	3	4	5	6	7	8
y	1.0	1.2	1.8	2.5	3.6	4.7	6.6	9.1

2. Find the correlation coefficient

X	65	66	67	67	69	71	72	70	65
Y	67	68	69	68	70	70	69	70	70

3. Fit an exponential curve $v = a e^{kt}$ for the following data:

t	0	2	4	6	8
v	150	63	28	12	5.6

4. a) Find the correlation coefficient

x	78	89	97	69	59	79	68	57
y	125	137	156	112	107	138	123	108

b) Find the rank correlation

x	6	4	9	8	1	2	3	10	5	7
y	1	6	5	10	3	2	4	9	7	8