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M. Sc. (Final) Examination, 2016

MATHEMATICS

Paper–I

(Topology & Functional Analysis)

Time : Three Hours

Maximum Marks : 100

PART - A (खण्ड-अ) [Marks : 20

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर पचास शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART - B (खण्ड-ब) [Marks : 50

Answer *five* questions (250 words each).

Selecting *one* from each unit. All questions carry equal marks.

प्रत्येक इकाई से **एक-एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART - C (खण्ड-स) [Marks : 30

Answer any *two* questions (300 words each).

All questions carry equal marks.

कोई **दो** प्रश्न कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART - A

UNIT - I

1\. (a) Determine the topology T for R generated by the class S of all closed intervals of the form [p, p+1] of length 1.

(b) Let U be the usual topology on R and $[1, 2] \subset R$ show that following sets are open relative to [1, 2]

(i) $\left[1, 1\frac{1}{2}\right[$

(ii) $\left[1\frac{3}{4}, 2\right[$

UNIT - II

(c) Show that every co-finite topological space is compact.

(d) Give an example of topological space (X, T) having a non-empty proper T-open subset Y such that Y is compact.

UNIT - III

(e) Show that space (R, S) is disconnecteed where S is the lower limit topology on R.

(f) Consider the topology :

$$T = \{\phi, \{a\} \{c, d\} \{a, c, d\} \{b, c, d, e\}, x\}$$

set $X = \{a, b, c, d, e\}$ and show that

(i) (X, T) is disconnected

(ii) Y = {b, d, e} is connected subset on X

UNIT - IV

(g) Define :

(i) Normed linear space

(ii) Banach space

(iii) Hahn-Banach theorem

(h) State open mapping theorem.

UNIT - V

(i) Define :

(i) Hilbert space

(ii) Orthogonal complement

(iii) Riesz representation theorem

(j) Define :

(i) Adjoint of an operator

(ii) Self adjoint operator

(iii) Normal operator

PART - B

UNIT - I

2. Let (X, T) be a topological space and let $Y \subset X$ then collection $T_y = \{G \cap Y : Y \in Z\}$ is a topology on Y. Prove.

3. State and prove lindelöf theorem.

UNIT - II

4. Closed subset of a compact sets are compact.

5. Show by means of an example that a compact subset of a non-hausdroff topological space need not be closed.

UNIT - III

6. If every two points of a subset E of a topological space X are contained in some connected subset of E. Then E is connected subset of X.

7. Prove complete regularity is hereditery property.

UNIT - IV

8. The linear space l_{∞}^{n} of all n-tuples $x = (x_1, \ldots x_n)$ of Scalars is a Banach space under the normed defined by :

$$\|x\|_{\infty} = \text{Max}\{|x_1|, |x_2| \ldots |x_n|\}$$

9. State and prove closed graph theorem.

UNIT - V

10. Find the linear transformation of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}_{2\times 3}$ relative to the basis

$\{(1,1),(0,1)\}$ of $R^2(R)$ and of

$\{(1,1,0)\,(0,1,1)\,(1,0,1)\}$ of $R^3(R)$.

11. State and prove Schwartz's inequality.

PART - C

UNIT - I

12. (a) Every second countable space is first countable.

(b) prove that every first countable space is not necessarily second countable.

(c) Every family of non-empty disjoint open subset of a second countable space is countable.

UNIT - II

13. (a) A topological space (X, T) is compact if and only if every collection of closed subset of X with F.I.P. is fixed i.e. has a non empty intersection.

(b) Every closed and bounded interval on a real line is compact.

UNIT - III

14. (a) A first countable space in which every convergent sequence has a unique limit is a Hausdorff space.

(b) Every regular Lindelöf space is normal.

UNIT - IV

15. (a) A linear space C[0, 1] of real valued continuous function on [0, 1] is normed space but not Banch space with the norm of a function $f \in C[0, 1]$ defined as

$$\|f\| = \int_0^1 |f(t)|dt$$

(b) Let N be a non-zero normed linear space then prove that N is Banach space $\Leftrightarrow \{x : \|x\| = 1\}$ is complete

UNIT - V

16. If the matrix of a linear transformation T on $V_3(C)$ w.r.t. the basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ is $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$

What is matrix of T w.r.t. the basis

$\{(0,1,-1)\,(1,-1,1)\,(-1,1,0)\}$