NIILM University

Preparatory Course in Mathematics

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Chapter-1

Perfect Numbers

The Pythagoreans produced a theory of numbers comprised of numerology and scientific speculation. In their numerology, even numbers were feminine and odd numbers masculine. The numbers also represented abstract concepts such as 1 stood for reason, 2 stood for opinion, 3 stood for harmony, 4 stood for justice, and so on. Their arithmetic had a theory of special classes of numbers. There were perfect numbers of two kinds. The first kind included only 10, which was basic to the decimal system and the sum of the first four numbers 1 + 2 + 3 + 4 = 10. The second kind of perfect numbers were those equal to the sum of their proper divisors.

A perfect number is a positive integer that is equal to the sum of it divisors. However, for the case of a perfect number, the number itself is not included in the sum. The Greeks called a number such as 6 or 28 a perfect number because the sum of the proper divisors in each case is equal to the number; the proper divisors of 6 are 1, 2, and 3, and their sum is 6.

Although perfect numbers are regarded as arithmetical curiosities, their study has helped to develop the theory of numbers. Euclid proved that a number n of the form (2n-1)*2n-1 is a perfect number if the factor 2n-1 is prime. For example, if n assumes the value 2, 3, 5, or 7, the expression 2n-1 takes on the value 3, 7, 31, or 127, all of which are prime. For these values of n we obtain the perfect numbers 6, 28, 496, and 8,128.

The Neoplatonists Nicomachus of Gerasa and Iamblichus of Chalcis listed these perfect numbers and concluded that they follow a pattern: They alternately end in a 6 or an 8, and there is one perfect number for each interval from 1 to 10, 10 to 100, 100 to 1,000 and 1,000 to 10,000. They conjectured that both parts of the pattern would continue, but in this they were wrong. The fifth perfect number, which was discovered in the fifth century, corresponds to n = 13 and is 33,550,336, with eight digits rather than six. In addition, the sixth perfect number, like the fifth, ends with a six.

In 1961, the twentieth perfect number was found. It contains 2,663 digits in the decimal representation and corresponds to the case where n = 4,423. Today, thirty-seven perfect numbers are known. The prime for the largest of these is 23,021,377, which is 909,526 digits in length, and the largest perfect number is 1,819,050 digits in length It is not known whether there are an infinite number of perfect numbers.

In 1757, Leonhard Euler probed that every even perfect number must be of Euclids form. It has also been proven that every even perfect number must end in six or eight and if it ends in six, the digit preceding it must be odd. No one has as yet discovered an odd perfect number, but it is known that none exist below 1020.

Square Numbers

Square array of dots, probably formed with pebbles, led the Greeks to numbers that were perfect squares- that is to numbers which, when expressed in a various of ways as the products of two numbers, would have two equal factors.

The most complete discussion of square numbers was given by a Greek, Nicomachus of Gerasa (c. A.D. 100) in Introdictio Arithmetica, the earliest extant manuscript, dating back to the tenth century. Nicomachus was not the original mathematician, but he did organize previous generations of mathematics in a clear and precise manner. The first 10 square numbers-

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Each is a result of multiplying a number by itself-

1*1, 2*2, 3*3, 4*4, 5*5

which can also be written-

12 , 22 , 32 , 42

Interesting facts:

If you add any 2 consecutive triangular numbers, you will always make a square number. The triangular numbers are formed, geometrically, like the square numbers

Triangular numbers can also be computed by the formula

n(n + 1)/2, for n = 1, 2, 3, 4, ...

As we see from the above figure, the first three triangular numbers are 1, 3, and 6.

All square numbers end in either a 0, 1, 4, 5, 6 or a 9. Never a 2, 3, 7, or an 8.

If you subtract 2 consecutive square numbers, you will get an odd number- 3, 5, 7, 9

The Nature of

What is (Pi)?

(Pi) is the ratio of the distance around a circle to the circles' diameter. Mostly (Pi) is written as a fraction:

(distance around a circle)/(distance across and through the center of a circle)

or more simply C/D

The result of this fraction is a famous nonending, nonrepeating infinite string of decimals that mathematicians have been studying ever since Archimedes (287-212 B.C.E.). Archimedes is credited with providing a method of calculating (Pi). This number goes something like this: 3.14159265358979323 and so on. Your calculator (or most calculators) go ten places. This is considered more than enough to ensure accuracy in measurements. Measuring is one of the main things you can do with this number.

What does this number mean?

What mathematicians discovered is the existence in nature of ratios. A ratio is a relationship between two things. For example 1/2 is a ratio that gives us information about how the 1 and 2 are related. In this case 1 is divided by 2. Not only do these relationships exist but some of them 'always' have the same answer. They don't change. In other words the ratio of a circles' diameter to its circumference is always the same. It doesn't matter what these two numbers are. Their ratio is always equal to that very long number with all the decimals.

What is (Pi) good for?

You can find the area of a circle, the circumference of a circle, and the volume of a cylinder using the value of (Pi). Remember finding the area of a quadrilateral is the length (l) times the width (w).

l x w

How do you account for shaded areas that are not square? Some math people call these shapes lunes. You can calculate area much easier by using your formula for (Pi): (1/2)*(Pi)*r2.

In this case the radius equals 1. The value for (Pi) is 3.14. The formula now looks like this:

(1/2)(3.14)(12) = area of the circle.

Prime Numbers

A natural number that possesses only two factors, itself and 1, is called a prime number. You may think that it would be fairly easy to figure out all of these numbers. Well, I am sure that you could find the prime numbers within the first 20 natural numbers without much trouble, and you may be able to go even further without the use of a calculator. However, it does become much harder to figure out all of the prime numbers, especially in your head.

It was an early Greek mathematician that is most famous for his work with prime numbers. His name was Eratosthenes. It was this Greek mathematician that created what is known as the Sieve of Eratosthenes. This Sieve allows anyone to find all of the prime numbers quite easily. This is how the Sieve works . . .

Eratosthenes wrote all of the natural numbers down and proceeded to sieve out all of the numbers that were not primes. The number one has only one factor, so it is not a prime number. We move to number two which is a prime number. Strike every second number after the number two (or multiple of two). Move to the next prime number. It is three. Now we strike every third number after the number three (or multiple of three) and so on. We continue to strike out every nth number after the number n.

Now you are probably wondering just how many prime numbers there are. Well, there are infinitely many primes. In other words, there are more than we could ever count. We know that there is an infinite number of primes because if you were to multiply all of the known primes together and add 1, then you would get a number that must be divisible by at least one new prime number. You can keep doing this forever and never find the end of the prime number list.

One more interesting thing about prime numbers. Prime numbers are considered the "building blocks" of the natural numbers because every single natural number, excluding the number 1, is either a prime number or a product of prime numbers. In other words, every number, for example, 160 is either a prime or can be factored into a product of primes. To factor 160, we observe that 160 is even and hence is divisible by 2 (which happens to be the only even prime), so we divide 160 by 2, giving 160 = 2*80. Now 80 is even and nece divisible by 2, so 80 = 2*40 and 160 = 2*2*40. But 40 is also divisible by 2, 40 = 2*20, and 160 = 2*2*2*20. Continuing to divide by 2 until we can no longer do so, we get 160 = 2*2*2*2*2*5. Now 5 is a prime, so we would write 160 as a product of primes like this: 2*2*2*2*2*5 = 25*5. If we were to multiply all of these prime numbers, the answer would be 160. Thus, since all numbers other than 1 are prime numbers or products of prime numbers, we can use a process like the above to find the prime divisors of

Amicable Numbers

Throughout history there have been many different interesting numbers or types of numbers. One of these types is amicable numbers. Amicable numbers are a pair of numbers with the following property: the sum of all of the proper divisors of the first number (not including itself) exactly equals the second number while the sum of all of the proper divisors of the second number (not including itself) likewise equals the first number.

For example let's show that 220 & 284 are amicable numbers:

First we find the proper divisors of 220:

1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110

If you add up all of these numbers you will see that they sum to 284.

Now find the proper divisors of 284:

1, 2, 4, 71, 142

These sum to 220, and therefore 220 & 284 are amicable numbers.

The set of 220 and 284 was the first known set of amicable numbers. Pythagoras discovered the relationship and coined the term amicable because he considered the numbers to be a symbol of friendship. No other pairs were known until 1636 when Fermat discovered 17,296 and 18,416 as a second pair. This pair was actually discovered over three hundred years earlier by the Arab mathematician al-Banna, but it was never known in the West until Fermat's findings. Then in 1638, Descartes discovered a third pair of 9,363,584 and 9,437,056.

The First Thirteen Amicable Pairs

1	220	284
2	1,184	1,210
3	2,620	2,924
4	5,020	5,564
5	6,232	6,368
6	10,774	10,856
7	12,285	14,595
8	17,296	18,416
9	63,020	76,084
10	66,928	66,992
11	67,095	71,145
12	69,615	87,633
13	79,750	88,730

It wasn't until 1747 when Leonhard Euler turned his attention to amicable numbers that progress began to take place. Euler was able to produce 58 pairs over the next three years. How did he find so many, so quickly? Euler developed a formula that would produce amicable pairs. The only problem was that the formula didn't generate every amicable pair. Today there are over 5000 known pairs, the largest of which was found by Mariano Garcia on October 4, 1997 contained 4829 digits in each pair.

There are still many questions left to be answered about amicable numbers. Are there an infinite amount of pairs? Do the pairs always appear so that they are both even or both odd? What about the existence of amicable triples or quadruples? These are all questions that mathematicians are searching & researching today. Who knows, maybe you could discover some of these answers and put yourself in the history books alongside Euler and Fermat.

The Golden Ratio: Phi

The ratio of 1:1.61803 has unique and storied history. It is also know as Phi, named after the Greek sculptor Phidias. The ratio is found in nature, art, architecture, poetry, music, and of course math.As stated, the Golden Ratio is also known as Phi. Phidias lived from c. 500 - 432 BC. He incorporated the ratio into many of his sculptures, one of which is the ancient Greek temple, the Parthenon. This was a rectangle shaped temple whose sides are in the Agolden proportion@.

The number Phi is as unique as the number pi. It is a number whose decimal never ends and never has a pattern repeat itself. Also, there are only two numbers that remain the same when you square them, 0 and 1. To fine the value of Phi squared you simple add one to Phi. There are only two numbers that have this property. It is Phi and a number closely related to it: 0.61803. This number is also known as phi (note the lower case p).

Phi is supposedly the >most pleasing= rectangle proportion. It is found in not only Greek architecture, but other architecture as well. A couple examples are the pyramids and the United Nations building.

The Golden Ratio is also seen in nature. It can be found in many different ways on the human body. For instance, measure from the top of the head to the base of the neck and also from the base of the neck to the belly button. You will find this ratio is close to the Golden Ratio. Other examples can be found in flowers, ferns, eggs, and sea shell to name a few.

It is found in art, also. Many books about painting will point out that it is better to position objects to Aabout one-third@ of the way across and not in the center of the picture. This seems to make the picture more pleasing to the eye.

Another interesting thing about the Golden Ratio has to do with the Fibonacci sequence. If you find the ratio of two successive numbers in the sequence you will find as the numbers get larger the ratio gets closer and closer the Phi. For instance, 5/3 = 1.667. 233/144 = 1.6181. 610/377 =1.6181.

Examples of the Golden Ratio can be seen everywhere. It is a unique number that has some very interesting properties. I have only touched on the surface of what is out there about it. Please check some of the web sites listed below to find out more about the Golden Ratio or Phi.

Contributed by James Means

Animals and Numbers

Can animals count or even have a concept of numbers? Many specialists in animal behavior have conducted experiments that have shown animals to possess an apparent sense of rudimentary perception of quantities. This is sometimes called the "number " sense. The number sense allows an animal to determine the differences in size between two small collections of similar objects. It also determines that a collection isn't the same after some objects are removed. Mothers of domestic animals and other animals have shown that they can definitely determine and perceive when one of their young is missing from the group.

Chapter-2

Why We Learn Mathematics

Birds have shown that they can be trained to determine the number of seeds in different piles of seeds up to the number five. An example of their ability to count up to five is contained in the story of the crow. In this story, a squire was very frustrated with a particular crow. This crow had made its' nest in the watchtower of the squire's estate. The squire continuously tried to surprise the crow, but had no such luck. Each time the squire approached the nest, the crow would leave. It would not return until the squire had left. From a distance the bird would watch for the man's departure. With this in mind, the squire came up with a plan to fool the bird. He took another man into the tower with him. Hoping to outwit the bird, one man left the tower. However, the bird would still not return to the tower until both men had departed. This experiment was repeated in the succeeding days with two, three, and even four men. Yet, the crow was not to be fooled. Finally, five men were sent into the tower. As before, all entered the tower. Only one remained as the other four left the area. At this point, the crow lost count and returned to it's nest. Not being able to distinguish above the number four cost the crow it's life, as the squire rid the tower of the crow forever.

Birds are not the only creatures with a concept of numbers. The behavior of insects, such as the solitary wasp, indicates some basic mathematical understanding. The mother wasp lays her eggs in individual cells and provides each egg with a number of live caterpillars, on which the young will feed on when they hatch. The number of caterpillars is different among species, but it is always the same for each sex of eggs. The male is smaller, so the mother supplies him with only five caterpillars. The female, who is large, receives ten caterpillars in her cell. This shows that the mother can not only distinguish which cell contains a male or a female, but she can also distinguish between the numbers five and ten in the caterpillars she is providing.

Other animals, such as animals in circuses, have been hypothesized to have mathematical talents. Animals such as horses and elephants are displayed to have the abilities to add, subtract, and count. However, these talents are more of a trained response triggered by a cue from the trainer. The skills demonstrated are not an innate response, as seen with the crow and the wasp.

Contributed by Lloyd Holt

Ideal & Irrational Numbers

The Pythagoreans were people who followed the mathematician, Pythagoras, best known for the right triangle theorem which is named after him. They believed that the universe could be explained by numbers. They were especially aware of the numbers 1,2,3,4 which they called the tetractys. They swore to the tetractys and saw "fourness" in many things. The geometric elements point, line, surface and solid are examples of fourness.

To Pythagoras, numbers had personalities - masculine or feminine, perfect or incomplete, beautiful or ugly. Ten was the best: containing one, two, three, and four - and written in dot formation it formed a perfect triangle. Because the sum of 1,2,3 and 4 is 10; ten was considered the ideal number.

Whereas four represented nature, they believed that ten represented the universe. They used the eight planets which had been discovered at this time plus two invented planets to support this belief. The Pythagoreans were unable to see these two planets because the part of the Earth where they lived "faced away from these planets."

Pythagoras looked at ratios of whole numbers to explain nature. Upon his death, his followers used the method of indirect proof to show that the square root of 2 cannot be written as a ratio of two whole numbers. In using this proof, irrational numbers - numbers which cannot be written as the ratio of two whole numbers - were discovered.

PROOF:

sqr (2) = m/n (a fraction reduced to lowest terms)

2 = m2/n2

m2 = 2n2 (so, m is a multiple of 2, call it 2q)

4q2 = 2n2

2q2 = n2 (so, n is a multiple of 2)

So, both m & n are multiples of 2, which is impossible, because m/n was reduced to lowest terms. Thus the proof that the square root of 2 cannot be expressed as a fraction, ie it is an irrational number.

Fibonacci Numbers

The Fibonacci Number Sequence was first presented in Leonardo Pisano's book, "Liber abaci" or "Book of Calculating". It is a sequence that I find to be very fascinating, and suprisingly it is a part of every day nature.

The Fibonacci sequence can be found in sea shell spirals, branching plants, petals on flowers, and in pine cones. I will explain this sequence to you in 3 different ways: the basic sequence, the rabbit problem, and the bees.

The Basic Sequence

The first twenty numbers of the sequence are as follows:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181

The numbers are obtained by adding two numbers to get the next. For example:

0 + 1 = 1	5 + 8 = 13	55 + 89 = 144	610 + 987 = 1597
1 + 1 = 2	8 + 13 = 21	89 + 144 = 233	987 + 1597 = 2584
1 + 2 = 3	13 + 21 = 34	144 + 233 = 377	1597 + 2584 = 4181
2 + 3 = 5	21 + 34 = 55	233 + 377 = 610	and so on
3 + 5 = 8	34 + 55 = 89	377 + 610 = 987	

The Rabbit Problem

This is another problem stated in Pisano's book. The question posed wanted you to figure out, under ideal circumstances, if you placed 2 rabbits together how many pairs of rabbits could be produced from the 1st pair in one year. Assume that the rabbits never die and that the female always produces one new pair (1 male/1 female) every month from the 2nd month on.

The Bees

This example is more realistic than that of the rabbits because there is a chance that they will have more than 2 offspring at a time. It is also possible that the offspring won't be a male and a female. The breeding pattern of bees is much more practical.

Many female worker bees are sterile. The queen bee's unfertilized eggs produce male bees. This means that they will have no "father". Female bees are produced when the queen actually mates with a male. They have both parents. Most females are workers but some are selected to become queen bees. From these facts we find the Fibonacci numbers:

The male bee has 1 parent

The female bee has 2 parents, 3 grand-parents, 5 great-grand-parents, 8 great-great-grand-parents, and 13 great-great-great-grand-parents

The History of Zero

In today's modern mathematics, we have become accustomed to zero as a number. It's hard to believe that most ancient number systems didn't include zero. The Mayan civilization may have been among the first to have a symbol for zero. The Mayas flourished in the Yucatan peninsula of Mexico about 1300 years ago. They used the as a placeholder, in a vertical place-value system. It is considered one of their cultures greatest achievements.

The ancient Egyptians, Romans, and Greeks alike had no symbol for zero. In Greek geometry, zero and irrational numbers were impossible. The Greeks made great strides in mathematics, but it was all done with a number system without zero. The Greek astronomer Ptolemy (ca. A.D. 150) was the first to write a zero at the end of a number. For this he used a circular symbol.

In ancient Babylonian history there was no use of the zero. In the later Babylonian or during the Seleucid period a special symbol, which was also used as a separation mark between sentences, came into use for a zero? There's a definite possibility that the Babylonians used this mark for a zero within a number, as early as the end of the eighth century B.C. Up until the time of Aristotle, there seems to be no evidence that the Babylonians ever regarded zero as a number. Aristotle discussed division by zero in connection with speed through a vacuum.

Throughout the Dark Ages, Western mathematics was held back by the Roman's traditional numbering system. The first to think differently was Leonardo Fibonacci. He was a merchant's son, born in the Italian city-state Pisa, late in the twelfth century. In Pisa, he studied the work of Euclid and other Greek mathematicians. When he was still a boy, he moved to the Muslim city of Bugia, in North Africa. There he examined leather and furs before they were shipped back to Pisa. Leonardo got an education in Arabic culture as he traveled around the Mediterranean to Constantinople, Egypt and Syria. He recognized that the Hindu-Arabic numerals, the numerals we use today, were superior to the Roman numerals he had grown up with in the West.

In the sixth century, mathematicians in India developed a place-value system. They introduced the concept of zero to keep their symbols in their proper places. In the seventh century, Hindu scholars introduced to Islam the ideas of zero and place-value. These ideas spread rapidly throughout the Arabic world. Six centuries later, Fibonacci was so impressed with the ease of Hindu-Arabic numerals that he wrote a book entitled Liber abaci.

The Pisan local merchants, the trading class, ignored Fibonacci book. They were wallowing in prosperity and did not want to be bothered with giving up Roman numerals and adopting a zero. Fibonacci mathematician friends liked the new number system and slowly over time gave up the

Roman numerals. By the fifteenth century, the numerals were showing up on coins and gravestones. Western mathematics had emerged from the Dark Ages, and was flourishing into a new number system with a zero, the Hindu-Arabic numerals. The immediate advances in mathematics after that time are proof of the importance of, the zero.

Contributed by Pam Nye

Why Learn Mathematics?

Technology is everywhere around us, and you need mathematics to master it!

In fact most top-paying jobs need good math skills:

- Doctors
- Vets
- Engineers
- Scientists
- Software Developers
- Marketing Analysts
- Financial Officers
- Investment Managers
- and more ...

And Mathematics is not just numbers, it is about patterns, too!

So jobs like fashion and interior design benefit from math skills.

Mathematics is also useful in everyday life:

- Investing money (interest rates, profits, etc)
- Estimating costs
- Shopping (is it really a bargain?)
- Understanding Computers
- Designing rooms and gardens
- Planning trips

And anyway, it is just plain fun: what other subject is about solving puzzles?

How to Learn

Learning has two major steps:

Get the information ... read, listen to a teacher, watch a video.

Use the information ... sketch it, think about it, answer questions.

Using it is so important! Answering questions helps you organize the ideas in your mind*.

How to Read Mathematics

Mathematics says a lot in a short space.

Example, in English: "We don't know what staplers or trays cost, but we do know that the office manager bought 15 staplers and 11 trays for a total cost of $73"

But in Mathematics: $15s + 11t = 73$

So it is good to re-read, go back and forth and play with the ideas.

Reading Mathematics is different to reading English

Read it, think about it, read again, write it down or sketch it out, and then use it (by answering questions), that all helps to get the ideas into your mind.

Example: Converting Celsius to Fahrenheit

$°F = (°C \times 9/5) + 32$

- Read it once first to see there is °F (which means Fahrenheit) on one side, and °C(Celsius) on the other side, with some calculations.
- Now go over it again and see that °C is multiplied by 9/5 and think "I wonder why that is done? Why 9/5?" Then notice that 32 is added ... why is that?
- Maybe you could make a sketch (as shown below)
- Then try using it yourself, do a few conversions and see how it works

Make Sketches

It really helps you to understand if you try sketching what you are learning*.

Make large and bold sketches with plenty of labels and notes.

Like this sketch about Celsius and Fahrenheit:

Work Neatly

Working neatly helps you think more clearly

and also gives you good mental habits.

Have pride in your work, even if it may go into the bin later.

Take Your Time!

Math is not about reading pages ... it is about building concepts in your mind.

So don't think "I read 2 pages today", instead think "I understand graphs better now".

It is important to learn about one idea at a time, make sure you understand it, and do plenty of exercises so you become expert.

Important: If you skip past a section, the rest may not make sense.

You will get confused, frustrated, and you will begin to hate the subject.

The cure?

- Go back to wherever it made sense,
- then move gently forward again,
- do plenty of practical things like solving questions and doing sketches

And you will soon be "back on track"

Read a Lot

Get some books, and read them. Spend time on math websites (like this one!), and join a forum (like the Math is Fun Forum).

Come Up With Your Own Ways

You have your own learning style.

Don't just follow the steps you are shown, try your own ideas!

Play with the ideas you are learning.

And try reading about the same subject from different places, you may find some that make a lot more sense to you.

Your mind is an amazing and unique tool, and you want to use it the best way you know how.

And studying mathematics is a good way to improve it!

All About Ideas

It is more important to know the ideas than to remember the formulas.

If you know how things work, you can always re-create the formulas when you need them. And you may also be able to do more clever things with your ideas.

Chapter-3

The World of Numbers

Every subject contains information necessary to become a knowledgeable and functional member of our society. As we become more technologically dependent, technical reasoning is needed for survival. Mathematics is no longer just a subject taken by the elite. Now it has rightfully become a staple in our educational systems even though it is not appreciated by many people until it is needed. I decided to write this essay to help people become open to learning math by understanding what math is all about. [Hidden Agenda: Since I am a high school math teacher, this essay allows me to spend more time teaching mathematical techniques instead of talking about this during class.

What is math?

Those who do not appreciate math are those who do not understand what math is all about. That is why the nature of math desperately needs to be explained. Simply put, math is about solving problems.

How can math help me solve problems?

Ever since there were humans in existence, there have been problems to solve. Whether the problems were over basic requirements like sustaining sufficient amounts of food or major accomplishments like constructing multifunctional homes, problems such as these remain with us to this day. The peculiar thing about problems is that they all have similar properties.

What do all problems have in common?

Successful problem solvers are able to understand what is expected of the problems they face. In other words, they know all of the details surrounding the problem at hand, which is the most important step to solving problems. It requires an attention to detail and therefore patience. After examining the details, intelligent choices need to be made as well as the beginning steps of developing a strategy. The plan must be carried out in an order that makes sense. So careful planning, possibly by justifiable experimentation, must take place. Once an actual solution is obtained, it must be tested to determine whether or not it is reasonable.

What does problem solving have to do with math in school?

Every math problem that gets discussed, handled, and assigned forces us to use many, if not all, of the detailed methods of problem solving. Each individual problem becomes a small but important lesson for solving problems in general. Math is traditionally learned by first doing many smaller problems. Then the small problems are put together to solve bigger problems. For instance, in order to solve algebraic equations, being knowledgeable about addition, subtraction, multiplication, and division is a must. Ordering the steps to be carried out, evaluating expressions, and learning how and when equations are used must be learned, too.

Who commonly uses math?

Everybody uses math whether they realize it or not. Shoppers use math to calculate change, tax, and sales prices. Cooks use math to modify the amount a recipe will make. Vacationers use math to find time of arrivals and departures to plan their trips. Even homeowners use math to determine the cost of materials when doing projects.

Which professions use math?

Here is a small list of math orientated careers:

Accountants assist businesses by working on their taxes and planning for upcoming years. They work with tax codes and forms, use formulas for measuring interest, and spend a considerable amount of energy organizing paperwork.

Agriculturists determine the proper amounts of fertilizers, pesticides, and water to produce bountiful foods. They must be familiar with mixture problems.

Architects design buildings for structural integrity and beauty. They must know how to calculate loads for finding acceptable materials in design.

Biologists study nature to act in concert with it since we are so closely tied to nature. They use proportions to count animals as well as use statistics/probability.

Chemists find ways to use chemicals to assist us which entails purifying water, dealing with waste management, researching superconductors, analyzing crime scenes, making food products, ...

Computer Programmers create complicated sets of instructions called programs/software to help us use computers to solve problems. They must have strong logic skills.

Engineers (Chemical, Civil, Electrical, Industrial, Material) build products/structures/systems like automobiles, buildings, computers, machines, and planes, to name just a few examples. They cannot escape the frequent use of calculus!

Geologists use mathematical models to find oil and study earthquakes.

Lawyers argue cases using complicated lines of reason. That skill is nurtured by high level math courses. The also spend a lot of time researching cases.

Managers maintain schedules, regulate worker performance, and analyze productivity.

Medical Doctors must understand the dynamic systems of the human body. They research illnesses, carefully administer the proper amounts of medicine, read charts/tables, and organize their workload.

Meteorologists forecast the weather for agriculturists, pilots, vacationers, and those who are marine dependent.

Military Personnel carry out a variety of tasks ranging from aircraft maintenance to following detailed procedures.

Nurses carry out the detailed instructions doctors give them. They adjust intravenous drip rates, take vitals, dispense medicine, and even assist in operations, .

Politicians help solve the social problems of our time by making complicated decisions.

Technicians repair and maintain the technical gadgets we depend on like computers, TV's, VCR's, cars, refrigerators, ... They are always reading measuring devices, referring to manuals, and diagnosing system problems.

Tradesmen (carpenters, electricians, mechanics, and plumbers) estimate job costs and use technical math skills specific to their field. They deal with slopes, areas, volumes, distances and must have an excellent foundation in math.

Can I get a good job without learning a lot of math?

In all honesty, anything is possible. However, less and less labor intensive jobs are available. Workers in those fields are being replaced by machinery and robotics. Even when those jobs are available, the pay is usually substandard. In order to gain successful employment, technical skills must be learned. Someone has to fix all of those machines and robots.

What are employers looking for?

Employers are looking for three basic traits. They want their employees to be able to reason, work with technical equipment, and communicate their thoughts with other employees. It is clear that math deals with developing reason and working with technical equipment. It is not so clear how math affects communication. Successfully using math can improve the ability to speak and write more clearly. Language, at least the type needed for work, tends to be extremely structured and mathematical ability helps deal with that structure.

After high school, what do I do to learn more maths?

Basically, there are four avenues of education to pursue: universities, community colleges, trade schools, or the military.

Universities prepare students for highly professional careers. Math is typically a strong component of their curricula due to the extreme technical nature of these professions.

Community Colleges assist students to either go on to universities or learn technical skills needed for data processors, electronic technicians, law enforcers, mechanics, nurses, and realtors. Math is not as intense compared to the universities but is integrated throughout each program.

Trade schools teach students the science of automotive maintenance, carpentry, computer repair, heating and air-conditioning, plumbing, ... Math related skills are integrated throughout each program.

Even the military puts their people through school after basic training. These military schools are akin to trade schools. On the other hand, military officers, even though they must already have bachelor degrees, are put through further schooling after basic training.

As a general rule, I try to stay away from the front lines of the math wars, having always felt that the real wisdom about math is to be found in the no man's land between the two opposing camps. But President Bush's call for more and better math teachers in his recent State of the Union address led my local newspaper, the San Jose Mercury News, to ask me to pen an Op Ed on the subject. This I duly did, and my piece appeared on Sunday February 19. Attacks (fairly mild, I have to say) predictably followed from both sides, leading me to believe that my remarks probably came out more or less as I intended them to, as a call for reason. In any event, I survived my brief skirmish into dangerous waters sufficiently to be tempted to stick my foot into the pond once again. So here goes.

Much of my rationale for believing that the way forward in math ed is to be found in the DMZ of the war comes from my recognition that on both sides you find significant numbers of smart, well-educated, well-meaning people, who genuinely care about mathematics education. Unless you are of the simplistic George W. mindset, which the world divides cleanly into righteous individuals on the one side and “evil doers” on the other, it follows, surely, that both sides have something valuable to say. ("Evil doers" always stuck me as a strange phrase to hear uttered in public by a grown-up, by the way. It sounds more like the box-copy description of a band of orcs in a fantasy video game.) The challenge, then, is to reconcile the two views.

In brief, the gist of my Mercury News opinion piece was this. While it sounds reasonable to suggest that understanding mathematical concepts should precede (or go along hand-in-hand with) the learning of procedural skills (such as adding fractions or solving equations), this may be (in practical terms, given the time available) impossible. The human brain evolved into its present state long before mathematics came onto the scene, and did so primarily to negotiate and survive in the physical world. As a consequence, our brains do not find it easy to understand mathematical concepts, which are completely abstract. (This is part of the theme I pursued in my book The Math Gene, published in 2000.)

However, although we are not "natural born mathematicians," we do have three remarkable abilities that, taken together, provide the key to learning math. One is our language ability - our capacity to use symbols to represent things and to manipulate those symbols independently of what they represent. The second is our ability to ascribe meaning to our experiences - to make sense of the world, if you like. And the third is our capacity to learn new skills.

When we learn a new skill, initially we simply follow the rules in a mechanical fashion. Then, with practice, we gradually become better, and as our performance improves, our understanding grows. Think, for example, of the progression involved in learning to play chess, to play tennis, to ski, to drive a car, to play a musical instrument, to play a video game, etc. We start by following rules in a fairly mechanical fashion. Then, after a while, we are able to follow the rules proficiently. Then, some time later, we are able to apply the rules automatically and fluently. And eventually we achieve mastery and understanding. The same progression works for mathematics, only in this case, as mathematics is constructed and carried out using our language capacity, the initial rule-following stuff is primarily cognitive-linguistic.

Of course, there is plenty of evidence to show that mastery of skills without understanding is shallow, brittle, and subject to rapid decay. Understanding mathematical concepts is crucially important to mastering math. The question is: What does it take to achieve the necessary conceptual understanding, and when can it be acquired? Certainly my own experience is that conceptual understanding in mathematics comes only after a considerable amount of procedural practice (much of which therefore is of necessity carried out without understanding). How many of us professional mathematicians aced our high school or college calculus exams but only understood what a derivative is after we had our Ph.D.s and found ourselves teaching the stuff?

In fact, I can't imagine how one could possibly understand what calculus is and how and why it works without first using its rules and methods to solve a lot of problems. Likewise for most other areas of mathematics. In fact, the only parts of mathematics that I find sufficiently close to the physical and social world our brains developed to handle that there are innate meanings we could tap into are positive integer addition and subtraction for fairly small numbers, and perhaps also some fairly simple cases of division for small positive integers.

Interestingly, those were the only examples cited by the readers of my Mercury News article who argued against my suggestion that understanding comes only after a lot of procedural practice. Now it may be that in those particular areas, understanding can precede, or accompany the acquisition of, procedural mastery. Personally I doubt it, but I have yet to see convincing evidence either way. But, leaving those special (albeit important) cases aside, what about the rest of mathematics? Here I see no uncertainty. Understanding can come only after procedural mastery.

For example, physics and engineering faculty at universities continually stress that what they want their incoming students to have above all is procedural mastery of mathematics as a language - it is, after all, the language of science, as Galileo observed - and the ability to use various mathematical tools and methods to solve problems that arise in physics and engineering. Since even first-year physics and engineering involve use of tools such as partial differential equations, there is no hope that incoming students can have conceptual understanding of those tools and methods. But by a remarkable feature of the human brain, we can achieve procedural mastery without understanding. All it takes is practice. One of the great achievements of mathematics over the past few centuries has been the reduction of conceptually difficult issues to collections of rule-based symbolic procedures (such as calculus).

Thus, one of the things that high school mathematics education should definitely produce is the ability to learn and be able to apply rule-based symbolic processes without understanding them. Without that ability, progress into the sciences and engineering is at the very least severely hampered, and for many people may be cut off. (This, by the way, is the only rationale I can think of for teaching calculus in high school. Calculus is a supreme example of a set of rule-based procedures that can be mastered and applied without any hope of anything but the most superficial understanding until relatively late in the game. Basic probability theory and statistics are clearly far more relevant to everyday life in terms of content.)

Is mastery of rule-based symbolic procedures the only goal of school mathematics education? Of course not. The reason I am not focusing on conceptual issues is that much has been written on that issue - of particular note the National Research Council's excellent volume Adding it Up: Helping Children Learn Mathematics, published by the National Academy Press in 2001 (a book I have read from cover to cover on three separate occasions). My intention here is to shine as bright a light as possible on a mathematical skill that I think has, in recent years, been overlooked - and on occasion actively derided - by some in the math ed community. Life in today's society requires that we acquire many skills without associated understanding - driving a car, operating a computer, using a VCR, etc. Becoming a better driver, computer user, etc. often requires understanding the technology (and perhaps also the science behind it). But from society's perspective (and in many cases the perspective of the individual), the most important thing is the initial mastery of use. If something has been so well designed or developed that proficient use can be acquired without conceptual understanding, then the rapid acquisition of that skillful use is often the most efficient - and sometimes the only - way for an individual to move ahead. I think this is definitely the case with mathematics. I believe we owe it to our students to prepare them well for life in the highly technological world they will live in. In the case of mathematics, that means that one ability we should equip them with (not the only one by any means - Adding It Up lists several others, for instance) is being able to learn and apply rule-based symbolic processes without understanding them. That does not mean we should not provide explanations. Indeed, as a matter of intellectual courtesy, I think we should. But we need to acknowledge, both to ourselves and our students that understanding can come only later, as an emergent consequence of use. (No shame in that. It took 300 years from Newton's invention of calculus to a properly worked out conceptual basis for its rules and methods.)

Chapter-4

Fractions

Fractions

A circle is a geometric shape that we have seen in other lessons. The circle to the left can be used to represent one whole. We can divide this circle into equal parts as shown below.

This circle has been divided into 2 equal parts.

This circle has been divided into 3 equal parts.

This circle has been divided into 4 equal parts.

We can shade a portion of a circle to name a specific part of the whole as shown below.

Definition: A fraction names part of a region or part of a group. The top number of a fraction is called its numerator and the bottom part is its denominator.

So a fraction is the number of shaded parts divided by the number of equal parts as shown below:

Number of shaded parts numerator

Number of equal parts denominator

Looking at the numbers above, we have:

There are two equal parts, giving a denominator of 2. One of the parts is shaded, giving a numerator of 1.

There are three equal parts, giving a denominator of 3. Two of the parts are shaded, giving a numerator of 2.

There are four equal parts, giving a denominator of 4. One of the parts is shaded, giving a numerator of 1.

Note that the fraction bar means to divide the numerator by the denominator. Let's look at some more examples of fractions. In examples 1 through 4 below, we have identified the numerator and the denominator for each shaded circle. We have also written each fraction as a number and using words.

Why is the number written as three-fourths? We use a hyphen to distinguish a fraction from a ratio. For example, "The ratio of girls to boys in a class is 3 to 4." This ratio is written a 3 to 4, or

3:4. We do not know how many students are in the whole class. However, the fraction is written as three-fourths (with a hyphen) because 3 is 3/4 of one whole. Thus a ratio names a relationship, whereas, a fraction names a number that represents the part of a whole. When writing a fraction, a hyphen is always used.

It is important to note that other shapes besides a circle can be divided in equal parts. For example, we can let a rectangle represent one whole, and then divide it into equal parts as shown below.

Two equal parts

Three equal parts

Four equal parts

Five equal parts

Remember that a fraction is the number of shaded parts divided by the number of equal parts. In the example below, rectangles have been shaded to represent different fractions.

Example 5

One-half

One-third

One-fourth

One-fifth

The fractions above all have the same numerator. Each of these fractions is called a unit fraction.

Definition: A unit fraction is a fraction whose numerator is one. Each unit fraction is part of one whole (the number 1). The denominator names that part. Every fraction is a multiple of a unit fraction.

In examples 6 through 8, we will identify the fraction represented by the shaded portion of each shape.

Example 6

In example 6, there are four equal parts in each rectangle. Three sections have been shaded in each rectangle, but not the same three. This was done intentionally to demonstrate that any 3 of the 4 equal parts can be shaded to represent the fraction three-fourths.

Example

In example 7, each circle is shaded in different sections. However, both circles represent the fraction two-thirds. The value of a fraction is not changed by which sections are shaded.

Example 8

In example 8, each rectangle is shaded in different sections. However, both rectangles represent the fraction two-fifths. Once again, the value of a fraction is not changed by which sections are shaded.

In the examples above, we demonstrated that the value of a fraction is not changed by which sections are shaded. This is because a fraction is the number of shaded parts divided by the number of equal parts.

In example 9, the circle has been shaded horizontally; whereas, in example 10, the circle was shaded vertically. The circles in both examples represent the same fraction, one-half. The positioning of the shaded region does not change the value of a fraction.

In example 11, the rectangle is positioned horizontally; whereas in example 12, the rectangle is positioned vertically. Both rectangles represent the fraction four-fifths. The positioning of a shape does not change the value of the fraction it represents.

Remember that a fraction is the number of shaded parts divided by the number of equal parts

Classifying

Look at each fraction below. How are these fractions similar? How are they different?

The fraction is called a proper fraction. The fractions and are improper fractions.

Definition: A proper fraction is a fraction in which the numerator is less than the denominator.

Definition: An improper fraction is a fraction in which the numerator is greater than or equal to the denominator.

In example 1, we will identify each fraction as proper or improper. We will also write each fraction using words.

Example 1

Fraction	Type	Words
	improper	three-halves
	proper	two-fifths
	improper	three-thirds
	proper	five-sixths
	Improper	eleven-eighths
	Improper	eight-eighths

In example 2, each fraction has a numerator that is equal to its denominator. Each of these fractions is an improper fraction, equal to one whole (1). An improper fraction can also be greater than one whole, as shown in example 3.

In the improper fraction seven-fourths, the numerator (7) is greater than the denominator (4). We can write this improper fraction as a mixed number.

Definition: A mixed number consists of a whole-number part and a fractional part.

In examples 4 through 6, we will write each improper fraction as a mixed number.

In example 4, seven-fourths is an improper fraction. It is really the sum of four-fourths and three-fourths. Seven-fourths is written as the mixed number one and three-fourths, where one is the whole-number part, and three-fourths is the fraction

In example 5, the improper fraction seven-thirds is written as the mixed number two and one-third, where two is the whole-number part, and one-third is the fractional part.

Compare the numerator and denominator	Example	Type of Fraction	Write As
If the numerator < denominator, then the fraction < 1.		Proper fraction	proper fraction
If the numerator = denominator, then the fraction = 1.		Improper fraction	whole number
If the numerator > denominator, then the fraction > 1.		Improper fraction	mixed number

Fraction Vocabulary

Fractions - part of a number - i.e.   etc.

Numerator - top number of a fraction

Denominator - bottom number of a fraction

Proper Fraction - fraction with the lowest of the two numbers on the top - i.e.

Improper Fraction - fraction when the highest of the two numbers is on the top - i.e.

Greatest Common Factor(GCF) - the largest factor of two or more numbers.

Fraction in Lowest Terms - a fraction that has been reduced ,or divided so far that the numerator and the denominator do not share factors any longer.

Steps to finding GCF:

Write the factors of all your numbers

Look for all the numbers that are the same

The highest common number is the GCF

Example:Find the GCF of 16 and 24.

Solution: The GCF of 16 and 24 is 8.

Example: Put 4/8 into lowest terms. Solution - Divide both numbers by the greatest common factor.   In 4/8, the GCF is four, so after dividing both numbers by four, you get 1/2.

Least Common Multiple-the smallest number that is a multiple of two numbers.

Example- find the LCM of 4 and 6. Solution- Factors of 4 - 4,8,12,16,20,24,28... Factors of 6 - 6,12,18,24,30... Factors they have in common are 12 and 24. the lowest number is 12, so the LCM of 4 and 6 is 12 When adding fractions such as 2/4 and 1/4, all you have to do is add the numerators. (YOU NEVER ADD THE DENOMINATORS!) The denominators have to be the same when you add fractions. Since in this case, they are the same. just add the numerators. So 2/4+1/4=3/4.

However, many times, you will have to solve an addition problem such as 2/3+1/2. The denominators are not the same, so you can't add them just yet. Before you add, you have to find the Least Common Denominator or LCD. To find the LCD, all you have to do is find the LCM of the denominators.

Subtracting Fractions

To make this easy, let's just say that you use the same thought process as with adding fractions. You just look at the denominators, and if they are the same, then you proceed to subtract the first numerator by the second numerator. i.e: 6/5 - 4/5 = 2/5.

Mixed Numbers

Whole numbers that are mixed with fractions. If you have one whole and 3/4 remaining, the resulting fraction is 1 3/4. It is read one and three fourths. If you have a fraction like 8/5, you divide the numerator by the denominator. Five goes into eight once with 3/5 left over. Your resulting fraction is 1 3/5 and is read one and three fifths.

Adding Mixed Numbers

When you are adding fractions and they have unlike denominators, look at the denominators and turn it into a mixed fraction with like denominators. Add the fractions, and then the whole numbers.

Example: 4 1/2 + 2 1/3= 4 3/6 + 2 2/6

Solution: First, add the fractions which equal 5/6, then the whole numbers which equal 6. Your resulting fraction is 6 5/6.

Objectives

After completing this unit you should be able to:

Define algebra.

Recognize when an algebraic statement is an algebraic term, expression, or equation.

What is Algebra?

Algebra is a branch of mathematics that uses mathematical statements to describe relationships between things that vary over time. These variables include things like the relationship between supply of an object and its price. When we use a mathematical statement to describe a relationship, we often use letters to represent the quantity that varies, since it is not a fixed amount. These letters and symbols are referred to as variables. (See the Appendix One for a brief review of constants and variables.)

Chapter-5

Algebra

The mathematical statements that describe relationships are expressed using algebraic terms, expressions, or equations (mathematical statements containing letters or symbols to represent numbers). Before we use algebra to find information about these kinds of relationships, it is important to first cover some basic terminology. In this unit we will first define terms, expressions, and equations. In the remaining units in this book we will review how to work with algebraic expressions, solve equations, and how to construct algebraic equations that describe a relationship. We will also introduce the notation used in algebra as we move through this unit.

Algebraic Terms

The basic unit of an algebraic expression is a term. In general, a term is either a number or a product of a number and one or more variables. Below is the term –3ax.

The numerical part of the term, or the number factor of the term, is what we refer to as the numerical coefficient. This numerical coefficient will take on the sign of the operation in front of it. The term above contains a numerical coefficient, which includes the arithmetic sign, and a variable or variables. In this case the numerical coefficient is –3 and the variables in the term are a and x. Terms such as xz may not appear to have a numerical coefficient, but they do. The numerical coefficient is 1, which is assumed.

Algebraic Expressions

An expression is a meaningful collection of numbers, variables, and signs, positive or negative, of operations that must make mathematical and logical sense. Expressions:

contain any number of algebraic terms

use signs of operation—addition, subtraction, multiplication, and division.

do not contain an equality sign (=)

An example of an expression is:

–3ax + 11wx2y

In an expression, the signs of operation separate it into terms. The sign also becomes part of the term that it follows. The expression above contains two terms, the first term is –3ax and the second term is +11wx2y. The addition sign separates the two terms. For example, in the expression given above the plus sign (+) separates the –3ax from 11wx2y and is also part of the second term. Terms that do not have a sign listed in front of them are understood to be positive.

Below are several examples that are not expressions.

x + • y this statement tells us "x plus multiplied by y". This does not make mathematical or logical sense. This collection of symbols is nonsense.

y = 2x – this statement is not an expression because expressions are not allowed to contain the equal sign.

NOTE: The operation of multiplication can be represented by using a x, •, or by placing items to be multiplied in parentheses, brackets or braces, or in the case of variables, just written next to one another. The statements a xb, a • b, (a)(b), and ab are equivalent. In this booklet we will use the latter three representations.

Algebraic Equations

An equation is a mathematical statement that two expressions are equal. The following three statements are equations:

4 + 5 = 9 x – 35 = 56k2 + 3 x + 3 = 15

The first equation, 4 + 5 = 9, contains only numbers; the other two, however, also contain variables. All three contain two expressions separated by an equal sign:

When an equation contains variables you will often have to solve for one of those variables. Using equations to solve for a variable will be discussed later in this booklet.

.

The quadratic formula expresses the solution of the degree two equation in terms of its coefficients, .

Algebra (from Arabic al-jebr meaning "reunion of broken parts"is one of the broad parts of mathematics, together with number theory, geometry and analysis. Algebra arose from the idea that one can perform operations of arithmetic with non-numerical mathematical objects. At the beginning of algebra, and at elementary level, these objects are variables representing either numbers that are not yet known (unknowns) or unspecified numbers (indeterminates or parameters). This allows one to state and prove properties that are true no matter which specific numbers are involved. More generally, these objects may have various basic properties, and, presently, algebra is divided in several subareas which include linear algebra, group theory, ring theory and combinatorics (see below for more subareas).

Elementary algebra is the part of algebra that is usually taught in elementary courses of mathematics. Abstract algebra is a name usually given to the study of the algebraic structures (such as groups, rings, fields and algebras) themselves.

Algebra is also the name of various specific mathematical structures occurring in algebra. To distinguish between the meanings of the word, see below.

The adjective "algebraic" usually means relation to algebra, as in "algebraic structure". For historical reasons, it may also mean relation with the roots of polynomial equations, like in algebraic number, algebraic extension or algebraic expression. This comes from the fact that, until the end of 19th century, algebra was essentially the same area as the theory of equations. A witness of that is the fundamental theorem of algebra, which nowadays is not considered as belonging to algebra.

A mathematician who does research in algebra is called an algebraist.

For historical reasons, the word "algebra" has several related meanings in mathematics, as a single word or with qualifiers. Such a situation, where a single word has many meanings in the same area of mathematics, may be confusing. However the distinction is easier if one recalls that the name of a scientific area is usually singular and without an article and the name of a specific structure requires an article or the plural. Thus we have:

As a single word without article, "algebra" names a broad part of mathematics (see below).

As a single word with article or in plural, "algebra" denotes a specific mathematical structure. See algebra (ring theory) and algebra over a field.

With a qualifier, there is the same distinction:

Without article, it means a part of algebra, such as linear algebra, elementary algebra (the symbol-manipulation rules taught in elementary courses of mathematics as part of primary and secondary education), or abstract algebra (the study of the algebraic structures for themselves).

With an article, it means an instance of some abstract structure, like a Lie algebra or an associative algebra.

Frequently both meanings exist for the same qualifier, like in the sentence: Commutative algebra is the study of commutative rings, that all are commutative algebras over the integers.

Sometimes "algebra" is also used to denote the operations and methods related to algebra in the study of a structure that does not belong to algebra. For example algebra of infinite series may denote the methods for computing with series without using the notions of infinite summation, limits and convergence.

Algebra as a branch of mathematics

Algebra can essentially be considered as doing computations similar to that of arithmetic with non-numerical mathematical objectsInitially, these objects represented either numbers that were not yet known (unknowns) or unspecified numbers (in determinates or parameters), allowing one to state and prove properties that are true no matter which numbers are involved. For example, in the quadratic equation

are indeterminates and is the unknown. Solving this equation amounts to computing with the variables to express the un known in terms of the in determinates. Then, substituting any numbers for the indeterminates, gives the solution of a particular equation after a simple arithmetic computation.

As it developed, algebra was extended to other non-numerical objects, like vectors, matrices or polynomials. Then, the structural properties of these non-numerical objects were abstracted to define algebraic structures like groups, rings, fields and algebras.

Before the 16th century, mathematics was divided into only two subfields, arithmetic and geometry. Even though some methods, which had been developed much earlier, may be considered nowadays as algebra, the emergence of algebra and, soon thereafter, of infinitesimal calculus as subfields of mathematics only dates from 16th or 17th century. From the second half of 19th century on, many new fields of mathematics appeared, some of them included in algebra, either totally or partially.

It follows that algebra, instead of being a true branch of mathematics, appears nowadays, to be a collection of branches sharing common methods. This is clearly seen in the Mathematics Subject Classification where none of the first level areas (two digit entries) is called algebra. In fact, algebra is, roughly speaking, the union of sections 08-General algebraic systems, 12-Field theory and polynomials, 13-Commutative algebra, 15-Linear and multilinear algebra; matrix theory, 16-Associative rings and algebras, 17-Nonassociative rings and algebras, 18-Category theory;

homological algebra, 19-K-theory and 20-Group theory. Some other first level areas may be considered to belong partially to algebra, like 11-Number theory (mainly for algebraic number theory) and 14-Algebraic geometry.

Elementary algebra is the part of algebra that is usually taught in elementary courses of mathematics.

Abstract algebra is a name usually given to the study of the algebraic structures themselves.

Etymology

The word algebra comes from the Arabic language (الجبر al-jabr "restoration") from the title of the book Ilm al-jabr wa'l-muḳābala by al-Khwarizmi. The word entered the English language during Late Middle English from either Spanish, Italian, or Medieval Latin. Algebra originally referred to a surgical procedure, and still is in Spanish, while the mathematical sense was a later development.

History

The start of algebra as an area of mathematics may be dated to the end of 16th century, with François Viète's work. Until the 19th century, algebra consisted essentially of the theory of equations. In the following, "Prehistory of algebra" is about the results of the theory of equations that precede the emergence of algebra as an area of mathematics.

Prehistory of algebra

The roots of algebra can be traced to the ancient Babylonians who developed an advanced arithmetical system with which they were able to do calculations in an algorithmic fashion. The Babylonians developed formulas to calculate solutions for problems typically solved today by using linear equations, quadratic equations, and indeterminate linear equations. By contrast, most Egyptians of this era, as well as Greek and Chinese mathematics in the 1st millennium BC, usually solved such equations by geometric methods, such as those described in the Rhind Mathematical Papyrus, Euclid's Elements, and The Nine Chapters on the Mathematical Art. The geometric work of the Greeks, typified in the Elements, provided the framework for generalizing formulae beyond the solution of particular problems into more general systems of stating and solving equations, although this would not be realized until mathematics developed in medieval Islam.

By the time of Plato, Greek mathematics had undergone a drastic change. The Greeks created a geometric algebra where terms were represented by sides of geometric objects, usually lines, that had letters associated with them Diophantus (3rd century AD) was an Alexandrian Greek mathematician and the author of a series of books called Arithmetica. These texts deal with solving algebraic equations and have led, in number theory to the modern notion of Diophantine equation.

Earlier traditions discussed above had a direct influence on Muhammad ibn Mūsā al-Khwārizmī (c. 780–850). He later wrote The Compendious Book on Calculation by Completion and Balancing, which established algebra as a mathematical discipline that is independent of geometry and arithmetic.

The Hellenistic mathematicians Hero of Alexandria and Diophantus as well as Indian mathematicians such as Brahmagupta continued the traditions of Egypt and Babylon, though Diophantus' Arithmetica and Brahmagupta's Brahmasphutasiddhanta are on a higher level. For example, the first complete arithmetic solution (including zero and negative solutions) to quadratic equations was described by Brahmagupta in his book Brahmasphutasiddhanta. Later, Arabic and Muslim mathematicians developed algebraic methods to a much higher degree of sophistication. Although Diophantus and the Babylonians used mostly special ad hoc methods to solve equations, Al-Khwarizmi contribution was fundamental. He solved linear and quadratic equations without algebraic symbolism, negative numbers or zero, thus he has to distinguish several types of equations.

In the context where algebra is identified with the theory of equations, the Greek mathematician Diophantus has traditionally been known as the "father of algebra" but in more recent times there is much debate over whether al-Khwarizmi, who founded the discipline of al-jabr, deserves that title instead. Those who support Diophantus point to the fact that the algebra found in Al-Jabr is slightly more elementary than the algebra found in Arithmetica and that Arithmetica is syncopated while Al-Jabr is fully rhetorical. Those who support Al-Khwarizmi point to the fact that he introduced the methods of "reduction" and "balancing" (the transposition of subtracted terms to the other side of an equation, that is, the cancellation of like terms on opposite sides of the equation) which the term al-jabr originally referred to, and that he gave an exhaustive explanation of solving quadratic equations, supported by geometric proofs, while treating algebra as an independent discipline in its own right. His algebra was also no longer concerned "with a series of problems to be resolved, but an exposition which starts with primitive terms in which the combinations must give all possible prototypes for equations, which henceforward explicitly constitute the true object of study". He also studied an equation for its own sake and "in a generic manner, insofar as it does not simply emerge in the course of solving a problem, but is specifically called on to define an infinite class of problems".

The Persian mathematician Omar Khayyam is credited with identifying the foundations of algebraic geometry and found the general geometric solution of the cubic equation. Another Persian mathematician, Sharaf al-Dīn al-Tūsī, found algebraic and numerical solutions to various cases of cubic equations. He also developed the concept of a function. The Indian mathematicians Mahavira and Bhaskara II, the Persian mathematician Al-Karaji, and the Chinese mathematician Zhu Shijie, solved various cases of cubic, quartic, quintic and higher-order polynomial equations using numerical methods. In the 13th century, the solution of a cubic equation by Fibonacci is representative of the beginning of a revival in European algebra. As the Islamic world was declining, the European world was ascending. And it is here that algebra was further developed.

In 1545, the Italian mathematician Girolamo Cardano published Ars magna -The great art, a 40-chapter masterpiece in which he gave for the first time a method for solving the general cubic and quartic equations.

François Viète's work at the close of the 16th century marks the start of the classical discipline of algebra. In 1637, René Descartes published La Géométrie, inventing analytic geometry and introducing modern algebraic notation. Another key event in the further development of algebra was the general algebraic solution of the cubic and quartic equations, developed in the mid-16th century. The idea of a determinant was developed by Japanese mathematician Kowa Seki in the 17th century, followed independently by Gottfried Leibniz ten years later, for the purpose of solving systems of simultaneous linear equations using matrices. Gabriel Cramer also did some work on matrices and determinants in the 18th century. Permutations were studied by Joseph-Louis Lagrange in his 1770 paper Réflexions sur la résolution algébrique des équations devoted to solutions of algebraic equations, in which he introduced Lagrange resolvents. Paolo Ruffini was the first person to develop the theory of permutation groups, and like his predecessors, also in the context of solving algebraic equations.

Abstract algebra was developed in the 19th century, deriving from the interest in solving equations, initially focusing on what is now called Galois theory, and on constructibility issues. The "modern algebra" has deep nineteenth-century roots in the work, for example, of Richard

Dedekind and Leopold Kronecker and profound interconnections with other branches of mathematics such as algebraic number theory and algebraic geometry. George Peacock was the founder of axiomatic thinking in arithmetic and algebra. Augustus De Morgan discovered relation algebra in his Syllabus of a Proposed System of Logic. Josiah Willard Gibbs developed an algebra of vectors in three-dimensional space, and Arthur Cayley developed an algebra of matrices Topics containing the word "algebra"

Areas of mathematics:

Elementary algebra, the part of algebra that is usually taught in elementary courses of mathematics.

Abstract algebra, in which algebraic structures such as groups, rings and fields are axiomatically defined and investigated.

Linear algebra, in which the specific properties of linear equations, vector spaces and matrices are studied.

Commutative algebra, the study of commutative rings.

Computer algebra, the implementation of algebraic methods as algorithms and computer programs.

Homological algebra, the study of algebraic structures that are fundamental to study topological spaces.

Universal algebra, in which properties common to all algebraic structures are studied.

Algebraic number theory, in which the properties of numbers are studied from an algebraic point of view.

Algebraic geometry, a branch of geometry, in its primitive form specifying curves and surfaces by solutions of polynomial equations.

Algebraic combinatorics, in which algebraic methods are used to study combinatorial questions.

Many mathematical structures are called algebras:

Algebra over a field or more generally algebra over a ring.

Many classes of algebras over a field or over a ring have a specific name:

Associative algebra

Non-associative algebra

Lie algebra

Hopf algebra

C*-algebra

Symmetric algebra

Exterior algebra

Tensor algebra

In measure theory,

Sigma-algebra

Algebra over a set

In category theory

F-algebra and F-coalgebra

T-algebra

In logic,

Relational algebra, in which a set of finitary relations that is closed under certain operators.

Boolean algebra, a structure abstracting the computation with the truth values false and true. See also Boolean algebra (structure).

Heyting algebra

Elementary algebra

Algebraic expression notation:

1 – power (exponent)

2 – coefficient

3 – term

4 – operator

5 – constant term

x y c – variables/constants

Elementary algebra is the most basic form of algebra. It is taught to students who are presumed to have no knowledge of mathematics beyond the basic principles of arithmetic. In arithmetic, only numbers and their arithmetical operations (such as +, −, ×, ÷) occur. In algebra, numbers are often represented by symbols called variables (such as a, n, x, y or z). This is useful because:

It allows the general formulation of arithmetical laws (such as $a + b = b + a$ for all a and b), and thus is the first step to a systematic exploration of the properties of the real number system.

It allows the reference to "unknown" numbers, the formulation of equations and the study of how to solve these. (For instance, "Find a number x such that $3x + 1 = 10$" or going a bit further "Find a number x such that $ax + b = c$". This step leads to the conclusion that it is not the nature of the specific numbers that allows us to solve it, but that of the operations involved.)

It allows the formulation of functional relationships. (For instance, "If you sell x tickets, then your profit will be $3x - 10$ dollars, or $f(x) = 3x - 10$, where f is the function, and x is the number to which the function is applied".)

Polynomials

Chapter-6

Polynomial

A polynomial is an expression that is the sum of a finite number of non-zero terms, each term consisting of the product of a constant and a finite number of variables raised to whole number powers. For example, x2 + 2x − 3 is a polynomial in the single variable x. A polynomial expression is an expression that may be rewritten as a polynomial, by using commutativity, associativity and distributivity of addition and multiplication. For example, (x − 1)(x + 3) is a polynomial expression, that, properly speaking, is not a polynomial. A polynomial function is a function that is defined by a polynomial, or, equivalently, by a polynomial expression. The two preceding examples define the same polynomial function.

Two important and related problems in algebra are the factorization of polynomials, that is, expressing a given polynomial as a product of other polynomials that cannot be factored any further, and the computation of polynomial greatest common divisors. The example polynomial above can be factored as (x − 1)(x + 3). A related class of problems is finding algebraic expressions for the roots of a polynomial in a single variable.

Teaching algebra

It has been suggested that elementary algebra should be taught as young as eleven years old,[24] though in recent years it is more common for public lessons to begin at the eighth grade level (≈ 13 y.o. ±) in the United States

Since 1997, Virginia Tech and some other universities have begun using a personalized model of teaching algebra that combines instant feedback from specialized computer software with one-on-one and small group tutoring, which has reduced costs and increased student achievement.

Abstract algebra

Abstract algebra extends the familiar concepts found in elementary algebra and arithmetic of numbers to more general concepts. Here are listed fundamental concepts in abstract algebra.

Sets: Rather than just considering the different types of numbers, abstract algebra deals with the more general concept of sets: a collection of all objects (called elements) selected by property specific for the set. All collections of the familiar types of numbers are sets. Other examples of sets include the set of all two-by-two matrices, the set of all second-degree polynomials (ax2 + bx + c), the set of all two dimensional vectors in the plane, and the various finite groups such as the cyclic groups, which are the groups of integers modulo n. Set theory is a branch of logic and not technically a branch of algebra.

Binary operations: The notion of addition (+) is abstracted to give a binary operation, $*$ say. The notion of binary operation is meaningless without the set on which the operation is defined. For two elements a and b in a set S, $a * b$ is another element in the set; this condition is called closure. Addition (+), subtraction (-), multiplication (×), and division (÷) can be binary operations when defined on different sets, as are addition and multiplication of matrices, vectors, and polynomials.

Identity elements: The numbers zero and one are abstracted to give the notion of an identity element for an operation. Zero is the identity element for addition and one is the identity element for multiplication. For a general binary operator $*$ the identity element e must satisfy $a * e = a$ and $e * a = a$. This holds for addition as $a + 0 = a$ and $0 + a = a$ and multiplication $a \times 1 = a$ and $1 \times a = a$. Not all sets and operator combinations have an identity element; for example, the set of positive natural numbers (1, 2, 3, ...) has no identity element for addition.

Inverse elements: The negative numbers give rise to the concept of inverse elements. For addition, the inverse of a is written −a, and for multiplication the inverse is written a−1. A general two-sided inverse element a−1 satisfies the property that a ∗ a−1 = 1 and a−1 ∗ a = 1 .

Associativity: Addition of integers has a property called associativity. That is, the grouping of the numbers to be added does not affect the sum. For example: $(2 + 3) + 4 = 2 + (3 + 4)$. In general, this becomes $(a * b) * c = a * (b * c)$. This property is shared by most binary operations, but not subtraction or division or octonion multiplication.

Commutativity: Addition and multiplication of real numbers are both commutative. That is, the order of the numbers does not affect the result. For example: $2 + 3 = 3 + 2$. In general, this becomes $a * b = b * a$. This property does not hold for all binary operations. For example, matrix multiplication and quaternion multiplication are both non-commutative.

Groups

Group (mathematics)

See also: Group theory and Examples of groups

Combining the above concepts gives one of the most important structures in mathematics: a group. A group is a combination of a set S and a single binary operation ∗, defined in any way you choose, but with the following properties:

An identity element e exists, such that for every member a of S, e ∗ a and a ∗ e are both identical to a.

Every element has an inverse: for every member a of S, there exists a member a−1 such that a ∗ a−1 and a−1 ∗ a are both identical to the identity element.

The operation is associative: if a, b and c are members of S, then (a ∗ b) ∗ c is identical to a ∗ (b ∗ c).

If a group is also commutative—that is, for any two members a and b of S, a ∗ b is identical to b ∗ a—then the group is said to be abelian.

For example, the set of integers under the operation of addition is a group. In this group, the identity element is 0 and the inverse of any element a is its negation, −a. The associativity requirement is met, because for any integers a, b and c, (a + b) + c = a + (b + c)

The nonzero rational numbers form a group under multiplication. Here, the identity element is 1, since 1 × a = a × 1 = a for any rational number a. The inverse of a is 1/a, since a × 1/a = 1.

The integers under the multiplication operation, however, do not form a group. This is because, in general, the multiplicative inverse of an integer is not an integer. For example, 4 is an integer, but its multiplicative inverse is ¼, which is not an integer.

The theory of groups is studied in group theory. A major result in this theory is the classification of finite simple groups, mostly published between about 1955 and 1983, which separates the finite simple groups into roughly 30 basic types.

Semigroups, quasigroups, and monoids are structures similar to groups, but more general. They comprise a set and a closed binary operation, but do not necessarily satisfy the other conditions. A semigroup has an associative binary operation, but might not have an identity element. A monoid is a semigroup which does have an identity but might not have an inverse for every element. A quasigroup satisfies a requirement that any element can be turned into any other by either a unique left-multiplication or right-multiplication; however the binary operation might not be associative.

All groups are monoids, and all monoids are semigroups.

Examples

Set: Natural numbers N Integers Z Rational numbers Q (also real R and complex C numbers) Integers modulo 3: Z3 = {0, 1, 2}

Operation + × (w/o zero) + × (w/o zero) + − × (w/o zero) ÷ (w/o zero) + × (w/o zero)

Closed Yes Yes Yes Yes Yes Yes Yes Yes Yes Yes

Identity 0 1 0 1 0 N/A 1 N/A 0 1

Inverse N/A N/A −a N/A −a N/A 1/a N/A 0, 2, 1, respectively N/A, 1, 2, respectively

Associative Yes Yes Yes Yes Yes No Yes No Yes Yes

Commutative Yes Yes Yes Yes Yes No Yes No Yes Yes

Structure monoid monoid abelian group monoid abelian group quasigroup abelian group quasigroup abelian group abelian group (Z2)

Rings and fields

Main articles: ring (mathematics) and field (mathematics)

See also: Ring theory, Glossary of ring theory, Field theory (mathematics), and glossary of field theory

Groups just have one binary operation. To fully explain the behaviour of the different types of numbers, structures with two operators need to be studied. The most important of these are rings, and fields.

A ring has two binary operations (+) and (×), with × distributive over +. Under the first operator (+) it forms an abelian group. Under the second operator (×) it is associative, but it does not need to have identity, or inverse, so division is not required. The additive (+) identity element is written as 0 and the additive inverse of a is written as −a.

Distributivity generalises the distributive law for numbers, and specifies the order in which the operators should be applied, (called the precedence). For the integers $(a + b) \times c = a \times c + b \times c$ and $c \times (a + b) = c \times a + c \times b$, and × is said to be distributive over +.

The integers are an example of a ring. The integers have additional properties which make it an integral domain.

A field is a ring with the additional property that all the elements excluding 0 form an abelian group under ×. The multiplicative (×) identity is written as 1 and the multiplicative inverse of a is written as a^{-1}.

The rational numbers, the real numbers and the complex numbers are all examples of fields. Math 182Permutation

• An arrangement where order is important
is called a permutation.

• An arrangement where order is not
important is called combination.Seating Arrangement

Purpose: You are a photographer sitting a group in a row for pictures. You need to determine how many different ways you can seat the group.

Part One

1. There are three people to sit down in a row. Let the colors red, blue, and green represent the three people. If Red is the first to sit down, show all the possible seating arrangements. (Use colored pencils to show the different arrangements.)
2. If Blue is the first to sit down, show all these possible arrangements.
3. If Green is the first to sit down, show all these possible arrangements.
4. How many total possible seating arrangements are there for three people?Part Two
5. There are five people in the group. Let the colors red, blue, green, yellow, and purple represent the five people. How many people could sit down first for the picture?
6. If Red sits down, how many people are left to sit down?
7. If Blue sits down, how many people are left to sit down?
8. If Green sits down, how many people are left to sit down?
9. If Yellow sits down, how many people are left to sit down?
10. Multiply how many can sit down at each turn. How many possible seating arrangements are there for five people?Part Two

5. There are five people in the group. Let the colors red, blue, green, yellow,

and purple represent the five people. How many people could sit down first for the picture?

6. If Red sits down, how many people are left to sit down?

7. If Blue sits down, how many people are left to sit down?

8. If Green sits down, how many people are left to sit down?

9. If Yellow sits down, how many people are left to sit down?

10. Multiply how many can sit down at each turn. How many possible seating arrangements are there for five people?

P= 5x4x3x2x1 = 120 ways• A seating arrangement is an example of a permutation because the arrangement of the “n” objects is in a specific order. The order is important for a permutation.

• When the order does not matter, it is a combination, because you are only interested in the group.Extend:

. Twelve people need to be photographed, but there are only five chairs. (The rest of the people will be standing behind and their order does not matter.) How many ways can you sit the twelve people on the five chairs? Extend:

Twelve people need to be photographed, but there are only five chairs. (The rest of the people will be standing behind and their order does not matter.) How many ways can you sit the twelve people on the five chairs?

__ __ __ __ __Extend:

Twelve people need to be photographed, but there are only five chairs. (The rest of the people will be standing behind and

their order does not matter.) How many
ways can you sit the twelve people on the
five chairs?
12 x 11 x 10 x 9 x 8 = 95040 ways Permutation

• An arrangement where order is important
is called a permutation.

• Example: Mario, Sandy, Fred, and
Shanna are running for the offices of
president, secretary and treasurer. In how
many ways can these offices be filled?

4 x 3 x 2 = 24.

The offices can be filled 24 ways.Combination

• An arrangement where order is not
important is called combination.

• Example: Charles has four coins in his
pocket and pulls out three at one time.

Chapter-7

Permutation and Combination

permutation or a combination:

1. In how many ways can five books be arranged
on a book-shelf in the library?
2. In how many ways can three student-council
members be elected from five candidates?
3. Seven students line up to sharpen their pencils.
4. A DJ will play three CD choices from the 5
requests.• Determine if the situation represents a
permutation or a combination:
1. In how many ways can five books be arranged
on a book-shelf in the library? permutation
2. In how many ways can three student-council
members be elected from five candidates?
combination
3. Seven students line up to sharpen their pencils.
permutation
4. A DJ will play three CD choices from the 5
requests. combination• Find the number of events:
1. In how many ways can five books be arranged
on a book-shelf in the library?
P = 5! = 5x4x3x2x1 = 120 ways• Find the number of events:
1. In how many ways can five books be arranged
on a book-shelf in the library? = 120 ways
2. In how many ways can three student-council
members be elected from five candidates?

10 ways

6

60

3!

5 4 3

5 3

x x

C• Find the number of events:

3. Seven students line up to sharpen their pencils.

P=7! = 7x6x5x4x3x2x1 = 5040 ways

4. A DJ will play three CD choices from the 5

requests.• Find the number of events:

3. Seven students line up to sharpen their pencils.

P=7! = 7x6x5x4x3x2x1 = 5040 ways

4. A DJ will play three CD choices from the 5

requests.

10 ways

6

60

3!

5 4 3

5 3

x x

Fundamental Counting Principle.

If Act 1 can be performed in m ways,

and Act 2 can be performed in n ways

no matter how Act 1 turns out,

then the sequence Act 1 and Act 2

can be performed in m·n ways.• Example 1: Eight horses-Alabaster,

Beauty, Candy, Doughty, Excellente,

Friday, Great One, and High 'n Mighty- run

a race.

• In how many ways can the first three

finishers turn out?• Example 1: Eight horses-Alabaster,

Beauty, Candy, Doughty, Excellente,

Friday, Great One, and High 'n Mighty- run
a race.
• In how many ways can the first three
finishers turn out?
___ ___ _ __• Example 1: Eight horses-Alabaster,
Beauty, Candy, Doughty, Excellente,
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• In how many ways can the first three
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_8_ ___ ___• Example 1: Eight horses-Alabaster,
Beauty, Candy, Doughty, Excellente,
Friday, Great One, and High 'n Mighty- run
a race.
• In how many ways can the first three
finishers turn out?
_8_ x _7__ ___• Example 1: Eight horses-Alabaster,
Beauty, Candy, Doughty, Excellente,
Friday, Great One, and High 'n Mighty- run
a race.
• In how many ways can the first three
finishers turn out?
_8_ x _7_ x _6_• Example 1: Eight horses-Alabaster,
Beauty, Candy, Doughty, Excellente,
Friday, Great One, and High 'n Mighty- run
a race.
• In how many ways can the first three
finishers turn out?
_8_ x _7_ x _6_ = 336 ways• Solution:
Extending the Fundamental Counting
Principle to three acts, finishing first ("Act

1 ") can happen in 8 ways, there are then 7 ways in which finishing second ("Act 2") can occur, and finally there are 6 ways in which third place ("Act 3 ") can be filled, so the first three finishers could occur in 8·7·6 = 336 ways.• Example 2: How many ways can 10 tosses of a coin turn out?• Solution:

Each of the 10 acts can occur in two ways (H or T).

So there are 2^{10} = 1024 different sequences possible.• Example 3: Given a list of 5 blanks, in how many different ways can A, B, and C be placed into three of the blanks, one letter to a blank? (Two blanks will be empty.)• Solution: There are 5 choices of a blank for A, then 4 for B, and finally 3 for C. So there are 5·4·3 = 60 ways in which the three letters can be placed in the five blanks.• Order matters in spelling and numbers- RAT and TAR are different orders of the letters A, R, and T, and certainly have different meanings, as do 1234 and 4231. These are permutations.

• But many times order is not important, these are combinations. ABC, ACB, BAC, BCA, CAB, and CBA are six different permutations of the letters A, B, and C from the alphabet, but they represent just one combination. • Choose one combination of four different letters from the alphabet.

• How many permutations does this one

combination give?• Example 4: In how many different ways
could a committee of 5 people be chosen
from a class of 30 students?• 5 positions to be filled

— — — — —

• 30 people to chosen from
• 29 left to choose from, etc
• Therefore 30x29x28x27x26 = 17,100,720
permutations
• Divide out repeats of 5!
• So 142,506 combinations• Example 5: If the first chosen would be
chair, the next one vice-chair, then
secretary, and finally treasurer, in how
many different ways could a committee of
four be chosen?• Example 7: In a toss of 10 different
honest coins, what is the probability of
getting exactly 5 heads? • Example 8: In 10 tosses of a "loaded"
coin, with probability of heads = 0.7, what
is the probability of getting exactly 5
heads?

Combinations and Permutations

What's the Difference?

In English we use the word "combination" loosely, without thinking if the order of things is important. In other words:

"My fruit salad is a combination of apples, grapes and bananas" We don't care what order the fruits are in, they could also be "bananas, grapes and apples" or "grapes, apples and bananas", its the same fruit salad.

"The combination to the safe was 472". Now we do care about the order. "724" would not work, nor would "247". It has to be exactly 4-7-2.

So, in Mathematics we use more precise language:

If the order doesn't matter, it is a Combination.

If the order does matter it is a Permutation.

So, we should really call this a "Permutation Lock"!

In other words:

A Permutation is an ordered Combination.

To help you to remember, think "Permutation ... Position"

Permutations

There are basically two types of permutation:

Repetition is Allowed: such as the lock above. It could be "333".

No Repetition: for example the first three people in a running race. You can't be first and second.

1. Permutations with Repetition

These are the easiest to calculate.

When you have n things to choose from ... you have n choices each time!

When choosing r of them, the permutations are:

n × n × ... (r times)

(In other words, there are n possibilities for the first choice, THEN there are n possibilites for the second choice, and so on, multplying each time.)

Which is easier to write down using an exponent of r:

n × n × ... (r times) = nr

Example: in the lock above, there are 10 numbers to choose from (0,1,..9) and you choose 3 of them:

10 × 10 × ... (3 times) = 103 = 1,000 permutations

So, the formula is simply:

nr

where n is the number of things to choose from, and you choose r of them

(Repetition allowed, order matters)

2. Permutations without Repetition

In this case, you have to reduce the number of available choices each time.

For example, what order could 16 pool balls be in?

After choosing, say, number "14" you can't choose it again.

So, your first choice would have 16 possibilites, and your next choice would then have 15 possibilities, then 14, 13, etc. And the total permutations would be:

16 × 15 × 14 × 13 × ... = 20,922,789,888,000

But maybe you don't want to choose them all, just 3 of them, so that would be only:

16 × 15 × 14 = 3,360

In other words, there are 3,360 different ways that 3 pool balls could be selected out of 16 balls.

But how do we write that mathematically? Answer: we use the "factorial function"

The factorial function (symbol: !) just means to multiply a series of descending natural numbers.

Examples:

4! = 4 × 3 × 2 × 1 = 24

7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5,040

1! = 1

Note: it is generally agreed that 0! = 1. It may seem funny that multiplying no numbers together gets you 1, but it helps simplify a lot of equations.

So, if you wanted to select all of the billiard balls the permutations would be:

16! = 20,922,789,888,000

But if you wanted to select just 3, then you have to stop the multiplying after 14. How do you do that? There is a neat trick ... you divide by 13! ...

16 × 15 × 14 × 13 × 12 ...

= 16 × 15 × 14 = 3,360

13 × 12 ...

Do you see? 16! / 13! = 16 × 15 × 14

The formula is written:

where n is the number of things to choose from, and you choose r of them

(No repetition, order matters)

Examples:

Our "order of 3 out of 16 pool balls example" would be:

16! = 16! = 20,922,789,888,000 = 3,360

(16-3)! 13! 6,227,020,800

(which is just the same as: 16 × 15 × 14 = 3,360)

How many ways can first and second place be awarded to 10 people?

$$\frac{10!}{(10-2)!} = \frac{10!}{8!} = \frac{3,628,800}{40,320} = 90$$

(which is just the same as: 10 × 9 = 90)

Notation

Instead of writing the whole formula, people use different notations such as these:

Example: P(10,2) = 90

Combinations

There are also two types of combinations (remember the order does not matter now):

Repetition is Allowed: such as coins in your pocket (5,5,5,10,10)

No Repetition: such as lottery numbers (2,14,15,27,30,33)

1. Combinations with Repetition

Actually, these are the hardest to explain, so I will come back to this later.

2. Combinations without Repetition

This is how lotteries work. The numbers are drawn one at a time, and if you have the lucky numbers (no matter what order) you win!

The easiest way to explain it is to:

assume that the order does matter (ie permutations),

then alter it so the order does not matter.

Going back to our pool ball example, let us say that you just want to know which 3 pool balls were chosen, not the order.

We already know that 3 out of 16 gave us 3,360 permutations.

But many of those will be the same to us now, because we don't care what order!

For example, let us say balls 1, 2 and 3 were chosen. These are the possibilites:

Order does matter	Order doesn't matter
1 2 3	
1 3 2	
2 1 3	
2 3 1	
3 1 2	
3 2 1	1 2 3

So, the permutations will have 6 times as many possibilites.

In fact there is an easy way to work out how many ways "1 2 3" could be placed in order, and we have already talked about it. The answer is:

$3! = 3 \times 2 \times 1 = 6$

(Another example: 4 things can be placed in $4! = 4 \times 3 \times 2 \times 1 = 24$ different ways, try it for yourself!)

So, all we need to do is adjust our permutations formula to reduce it by how many ways the objects could be in order (because we aren't interested in the order any more):

That formula is so important it is often just written in big parentheses like this:

where n is the number of things to choose from, and you choose r of them

(No repetition, order doesn't matter)

It is often called "n choose r" (such as "16 choose 3")

And is also known as the "Binomial Coefficient"

Notation

As well as the "big parentheses", people also use these notations:

Example

So, our pool ball example (now without order) is:

16! = 16! = 20,922,789,888,000 = 560

3!(16-3)! 3!×13! 6×6,227,020,800

Or you could do it this way:

16×15×14 = 3360 = 560

3×2×1 6

So remember, do the permutation, then reduce by a further "r!"

... or better still ...

Remember the Formula!

It is interesting to also note how this formula is nice and symmetrical:

In other words choosing 3 balls out of 16, or choosing 13 balls out of 16 have the same number of combinations.

16! = 16! = 16! = 560
3!(16-3)! 13!(16-13)! 3!×13!

Pascal's Triangle

You can also use Pascal's Triangle to find the values. Go down to row "n" (the top row is 0), and then along "r" places and the value there is your answer. Here is an extract showing row 16:

1 14 91 364 ...

1 15 105 455 1365 ...

1 16 120 560 1820 4368 ...

1. Combinations with Repetition

OK, now we can tackle this one ...

Let us say there are five flavors of icecream: banana, chocolate, lemon, strawberry and vanilla. You can have three scoops. How many variations will there be?

Let's use letters for the flavors: {b, c, l, s, v}. Example selections would be

{c, c, c} (3 scoops of chocolate)
{b, l, v} (one each of banana, lemon and vanilla)
{b, v, v} (one of banana, two of vanilla)
(And just to be clear: There are n=5 things to choose from, and you choose r=3 of them.

Order does not matter, and you can repeat!)

Now, I can't describe directly to you how to calculate this, but I can show you a special technique that lets you work it out.

Think about the ice cream being in boxes, you could say "move past the first box, then take 3 scoops, then move along 3 more boxes to the end" and you will have 3 scoops of chocolate!

So, it is like you are ordering a robot to get your ice cream, but it doesn't change anything, you still get what you want.
Now you could write this down as (arrow means move, circle means scoop).

In fact the three examples above would be written like this:

{c, c, c} (3 scoops of chocolate):
{b, l, v} (one each of banana, lemon and vanilla):
{b, v, v} (one of banana, two of vanilla):
OK, so instead of worrying about different flavors, we have a simpler problem to solve: "how many different ways can you arrange arrows and circles"

Notice that there are always 3 circles (3 scoops of ice cream) and 4 arrows (you need to move 4 times to go from the 1st to 5th container).

So (being general here) there are r + (n-1) positions, and we want to choose r of them to have circles.

This is like saying "we have r + (n-1) pool balls and want to choose r of them". In other words it is now like the pool balls problem, but with slightly changed numbers. And you would write it like this:

where n is the number of things to choose from, and you choose r of them

(Repetition allowed, order doesn't matter)

Interestingly, we could have looked at the arrows instead of the circles, and we would have then been saying "we have r + (n-1) positions and want to choose (n-1) of them to have arrows", and the answer would be the same ...

So, what about our example, what is the answer?

$$\frac{(5+3-1)!}{3!(5-1)!} = \frac{7!}{3!\times4!} = \frac{5040}{6\times24} = 35$$

In Conclusion

Phew, that was a lot to absorb, so maybe you could read it again to be sure!

But knowing how these formulas work is only half the battle. Figuring out how to interpret a real world situation can be quite hard.

But at least now you know how to calculate all 4 variations of "Order does/does not matter" and "Repeats are/are not allowed".

Chapter-8

Basic Concepts in Geometry

1.1 Introduction

We start geometry with the simplest idea – a point. It is shown using a dot, which is labelled with a capital letter.

• A

The example above is the point A.

A straight line is a set of points next to each other. In maths, it carries on in both directions to infinity – even if we don't draw it, we think of it going on for ever in two opposite directions.

It is shown either using two points (labelled with capital letters) on a line with a double headed arrow, or using a single lower case letter.

B

A

l

The line AB, and the line l.

A plane is a flat surface extending in all directions to infinity. It has no thickness, so it is two dimensional. A plane has no size or shape, but it can be shown using a quadrilateral labelled with a single capital letter.

P

Some of the ideas about points, lines and planes can be given as axioms (statements given without proof).

1. An infinite number of lines can be drawn through any given point.
2. One and only one line can be drawn through two distinct points.
3. When two lines intersect they do so at only one point.

•

•1-2

1.2 Collinear and Coplanar

Any two points will always lie on the line drawn between them, but three points do not have to – if they do, they are called collinear points. It the three or more points are not on the same line, they are non collinear.

C R

B P Q m

A

l

Line l passes through points A, B and C, so A, B and C are collinear. It is impossible to draw a straight line through all three of P, Q and R, so they are noncollinear or just non linear.

Similarly, points and lines which lie in the same plane are called coplanar, otherwise they are called non-coplanar.

There are several special cases of lines. A line segment is the straight path between two points (e.g. C and D). It is named using its two endpoints. It is written CD and can be read as “segment CD”.

As said above, a line has no end points. It extends forever in both directions. If the points M and N are on the line, it can be written as MN and read as “line MN”.

A ray is part of a line with one end point. It is infinite in the other direction. It is written as RS which is read as “ray RS” where R is the end, and S is another point on the line.

Because a plane as defined above is infinite in all directions, it can contain any number of lines, rays, line segments and points.

(What geometric idea does a torch suggest?)

1.3 Here are some axioms about planes:

Axiom: A plane can be defined using a line and one point not on the line. By using the definition of a line, a plane can contain three non-collinear points. Conversely, through any three non collinear points there can be one and only one plane.

C P

A B

So line AB and point C are contained in the same plane P . A , B and C are three non-collinear points through which one plane, the plane P can pass.

Axiom: If two lines intersect, exactly one plane passes through both of the lines.

Q

m

Plane Q contains intersecting lines l and m.

Axiom: If two planes intersect their intersection is exactly one line.

l

P

Q

Planes P and Q intersect and their intersection is line l.

Axiom: If a line does not lie in a plane, but intersects it, their intersection is a point.

l

Point A is the intersection point of line l and plane P.

•1-4

Example 1

Draw three non-collinear points A, B and C on paper.

How many different lines can be drawn through pairs of points?

Name the lines.

Solution

Three lines can be drawn, AB , BC and AC

A

B

C

Example 2

D   C

A   B

S   R

P   Q

For the above figure, answer the following (assuming all angles are 90°):

a. Name the lines parallel to line AB .

b. Are line AQ and point R coplanar?   Why?

c. Are points A, S, B and R coplanar? Why?

d. Name three planes passing through A.

Solution

a. CD , SR and PQ .

b. Yes. Any line and a point outside it are coplanar.

c. Yes. Two parallel lines are always coplanar.

d. ABCD, ADSP and ADCB.

•

•

• 1-5

1.4 Line Segments

A line segment is a part of a line. It has a fixed length and consequently two end points. They are used to name the line segment

P Q

The line PQ stretches to infinity in both directions, but the line segment PQ is finite, and starts at P and stops at Q.

A line contains an infinite number of line segments. If two line segments have a common end point, they can be added.

P M R

For the above line, PM and MR are two segments. They have a common end point, M. Therefore, PM + MR = PR

The midpoint of a line segment is the point which is an equal distance from the two endpoints. If M is the midpoint of the line segment PR , then

PM + MR = PR and PM = MR ,

so PR = 2PM = 2MR.

Every segment has one and only one midpoint.

Exercise

Are the following statements true or false?

1. Any number of lines can pass through a single given point.
2. If two points lie in a plane the line joining them also lies in the same plane.
3. Any number of lines can pass through two given points.
4. Two lines can intersect in more than one point.
5. Two planes intersect to give two lines.
6. If two lines intersect only one plane contains both the lines.

7. A line segment has two end points and hence a fixed length.
8. The distance of the midpoint of a line segment from one end may or may not be equal to its distance from the other.

Vocabulary

point – a place in space, which has no size, it is infinitely small

line – a set of points stretching to infinity in both directions, but with no width.

plane – a mathematical flat surface in two dimensions, i.e. with no thickness

axiom – a logical/mathematical statement which is given without proof as it should be obvious

intersect – cross (like an intersection is American English for where roads cross)

collinear – three or more points on the same line

non-collinear – three or more points which are not on the same line

ray – a line with one fixed end, infinite in the other direction.

conversely – an argument or statement made the other way round from the original. BASIC GEOMETRY KEY CONCEPTS

You will need the following to do this course: Geometer's Sketchpad (GSP), a compass, a straight edge, a calculator, a protractor, colored pencils, tape and Algebra I course notes.

Unit 1: The Language and Symbols of Geometry

A. Unit 1 Introduction

The learner will investigate the language and symbols of geometry by utilizing the terms of points, lines, planes, segment, midpoints, rays, angles, angle pairs, and perpendicular bisectors, as well as analyze two-dimensional and three-dimensional figures and relate to life-related problems.

The learner will:

• find angle relationships such as vertical angles, linear pairs, complementary angles, and supplementary angles.

• identify relationships between and among points, lines, and planes, such as betweenness of points, midpoint, distance, collinear, coplanar.....

• find the intersection of lines, planes, and solids.

• connect geometric diagrams with algebraic representations.

• integrate constructions such as segments and angles, segment bisectors, angle bisectors.

• use relationships among one-and two-dimensional measures.

• represent geometric figures and properties using coordinates.

• connect the concepts of distance and midpoint to coordinate geometry.

B. Lesson 1: Points, Lines, Planes, and Space

Read the definitions of line, point, plane, collinear points, non-collinear, coplanar points, non-coplanar points, betweeness of points and segment.

Use the Sketchpad (GSP) software and complete the following exercise. Print out your work. If you're using the demo version, please draw on paper what you see on your GSP screen.

Procedure:

1. Select Segment Tool and draw segment AB.
2. Use Select Tool and the shift key to select both end points.
3. Click on the Display menu and choose show labels.
4. Select the segment and go to the Construct menu and choose point on object.

Again, use Display to show labels.

5. Select two of the points using the shift key.6. Choose the Measure menu and choose distance to measure each distance from A
to B, B to C and A to C.
7. Select the segments AC and BC using the shift key.
8. Choose the Measure menu and calculate to find the sum.
9. Drag point C along segment AB while holding down the shift key.
10. Go to Tool to record what you observed.
11. Go to the Edit menu and select all.
12. Go to the File menu and print.

Complete this problem. Given point D is between points T and R. TD = 3x + 2, DR = 2x + 1, and TR = 38. Find TD and RD. Show work. (Hint: Draw a diagram first.)

C. Lesson 2: Distance and Midpoint

Read the definitions for distance and midpoint.

Now, you will use those concepts and the distance and midpoint formulas to solve some problems.

Distance formula: ( ) ( )

2 2

d x x y y = 2 1 2 1

− + −

Midpoint formula:

1 2 1 2

,

2 2

x x y y

M

+ +

=

Examples: Given the coordinates of A and B, find the distance AB and the midpoint M of segment AB.

1. A: (7, 11) and B: (1, 3)

( ) ( )

2 2

d = 1 7 3 11 − + −

7 1 11 3

,

2 2

M

+ +

=

( ) ( )

2 2

d = −6 8 + −

8 14

,

2 2

M

=

d = + 36 64 M = (4, 7)

d = 100

d =10

So, AB = 10 and M = (4, 7)

2. A: (1, 2) and B: (4, 6)

( ) ( )

2 2

d = 4 1 6 2 − + −

1 4 2 6

,

2 2

M

+ +

=

( ) ( )

2 2

d = + 3 4

5 8

,

2 2

M

=

d = +9 16 M = (2.5, 4)

d = 25

d = 5

So, AB = 5 and M = (4, 7)Complete the following problems: given the coordinates of A and B, find AB, the
coordinates of M, and the midpoint of segment AB.

a) A: (4, 2) b) A: (-9, -1) c) A: (-2, 1)

B: (7, 0) B: (-6, -2) B: (5, -3)

AB = ___________ AB = ___________ AB = ___________

M: __________ M: __________ M: __________

Chapter-9

Mensuration

Perpendicular

Read the definitions for perpendicular lines, perpendicular bisectors, and midpoints.

Now you will construct the perpendicular bisector of a line segment. To do this you will need a compass and straight edge.

Procedure:

1. Draw a line segment.
2. Draw a circle that has one of the line segment endpoints as its center and whose radius is more than half the length of the line segment.3. Draw another circle with the other endpoint as its center and whose radius is more than half the length of the line segments.
4. Draw a line through the two points where the circles intersect. This is the perpendicular bisector.

(Where the perpendicular bisector and line segment meet is the midpoint.)

Then, complete the following constructions:

1. Draw a segment with your straight edge. Its length doesn't matter. Label the segment AB.
2. Construct a line perpendicular to segment AB that does NOT pass through the midpoint. Label the intersection point X.
3. Construct the perpendicular bisector for segment AB. Label the new intersection point Y.
4. Identify the midpoint of segment AB. Use a ruler to verify showing your work.

E. Lesson 5: Angle Pairs

Read the definitions of adjacent, congruent, complementary angles, supplementary angles, vertical angles and linear pairs. The following is a diagram of angle pairs, where line l and x are parallel.

The following are angle pairs and their relationships for two parallel lines intersected by a transversal.

∠ 1, ∠ 2, ∠ 7 and ∠ 8 are exterior angles.

∠ 3, ∠ 4, ∠ 5 and ∠ 6 are interior angles.

∠ 1 and ∠ 8, and ∠ 2 and ∠ 7 are alternate interior angles.

∠ 3 and ∠ 6, and ∠ 4 and ∠ 5 are alternate exterior angles.

∠ 1 and ∠ 5, ∠ 2 and ∠ 6, ∠ 3 and ∠ 7, and ∠ 4 and ∠ 8 are corresponding angles.

∠ 3 and ∠ 5, and ∠ 4 and ∠ 6 are consecutive interior angles.

Using the concepts above, complete the following activity for vertical angles and linear pairs. Using Sketchpad software to answer, print out and complete the worksheet from this activity.

Vertical Angles and Linear Pairs Using Sketchpad

Purpose:

To discover the relationships between pairs of vertical angles and between linear pairs.

Procedure:

1. Select the Line tool and construct Line AB.
2. Construct another line, making sure it intersects Line AB between Point A and Point B, and that Point C and Point D are on opposite sides of Line AB.
3. Select the Arrow tool and highlight both lines.
4. Select Construct, then Point of Intersection. 5. Your figure should now resemble the figure shown above.
6. We now want to measure some angles. To measure AEC, you select the Arrow tool, then select those three points in that order - A, E, C, - with the vertex letter of the angle in the middle. Once all three points are highlighted, select Measure, then Angle. Record this measurement in the space provided for that angle for intersection 1 in the chart on the next page.
7. Repeat this procedure to measure the following angles: CEB, BED, and DEA. Record these measurements in the chart for intersection 1.
8. Click and hold on either Point A, Point B, Point C, or Point D and drag it to a new location in order to create a new intersection. The only restriction to your movement is that Points A and B must remain on opposite sides of line CD and Points C and D must remain on opposite sides of Line AB, and that Point E remain between the other four points.

9. Your measurements will change as you move the point. Record these new measurements for intersection 2. Repeat this process to complete the chart.
Vertical Angles and Linear Pairs - Sketchpad Worksheet

D. Lesson 3: Angles Used in Parallel Lines.
Read the definitions of parallel lines, same side interior angles, alternate exterior angles, alternate interior angles, corresponding angles, vertical angles, linear pairs, congruent and supplementary.
Rules to know:
• If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.
• If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.
• If two parallel lines are cut by a transversal, then the same side interior angles are supplementary.
• If two parallel lines are cut by a transversal, then the corresponding angles have the same measure.
• If two coplanar lines are cut by a transversal and the corresponding angles have the same measure, then the lines are parallel.Now complete these questions:
E. Lesson 4: Slopes of Perpendicular and Parallel Lines
Read the definitions for parallel lines, perpendicular lines, slope, reciprocal, and inverse reciprocal.
Things to know about parallel and perpendicular lines:
• Two non-vertical lines are parallel if and only if they have the same slope.
• Lines that are parallel have the same slope.
• Lines that are perpendicular have slopes that are inverse reciprocals.Example: If line l has a slope of 10, what is the slope of line x if they are perpendicular?
Since the lines are perpendicular, we need to find the inverse reciprocal of the slope of line l. Line l has a slope of 10. The reciprocal of 10 is
1
10

and the inverse
(opposite) of
1
10
is -
1
10
. So, the slope of line x is -
1
10
.

Finish this lesson by answering these questions.

Given each pair of lines' slopes, state whether the lines are parallel, perpendicular, or neither.

1. 2, 2
2. -2, -1/2
3. -5/6, 6/5
4. -7, 1/7

State whether the pairs of lines determine by each set of points is parallel, perpendicular, or neither. Remember to first find the slope for each line. Finding the slope of a line is an Algebra I concept that you should have already had. If you have forgotten how to calculate slope, then go to an Algebra I course and review how to calculate slopes of lines given two points.

5. Line AB and line CD: A (1, 2); B (2, 3) and C (8, -1); D (7, -2).
6. Line EF and line GH: E (3, 8); F (-2, -9) and G (-2, 0); H (9, -5).
7. Line ST and line XY: S (-5, -2); T (-4, 1) and X (7, 6); Y (8, 3).

Triangles

Introduction

The learner will investigate the types of relationships of triangles using the properties that relate to the segments and angles of triangles including inequalities, perpendicular bisectors, altitudes, and medians, determine missing measures using diagrams and appropriately apply theorems/corollaries for equilateral, isosceles, scalene triangles and interior/exterior angle properties to solve problems.

The learner will:

• connect geometric diagrams with algebraic representations.

• integrate constructions such as segment bisectors, perpendiculars, and polygons.

• describe, draw, and construct two-dimensional figures.

• use angle and side relationships such as triangle inequalities, isosceles and equilateral triangle properties, altitude, and median.

B. Lesson 1: Angles in TrianglesRead the definitions for acute angle, obtuse angle, right angle, acute triangle, right
triangle, and obtuse triangle.

You are now to create a collage of the different types of triangles that are in your world. You will need at least three pictures for each of these triangles: acute triangle, right triangle, and obtuse triangle. These pictures can come from the Internet or magazines or your own photographs. After you have located your pictures, then display them on a poster board, appropriately labeled.

C. Lesson 2: Measuring Angles

This lesson will review measuring angles with a protractor. To measure an angle with a protractor, you need to place the measuring line with the center, usually a hole, on the vertex and a line of the angle. Then, look at the angle measurements on the protractor to read the angle measurement. Keep in mind that an acute angle is less than 90° and that an obtuse angle is greater than 90°.

Interior Angles

This lesson will introduce you to one of the most used theorems in geometry.

Theorem: The sum of the interior angles in any triangle is 180°.Examples: Using the following picture complete the problems.

1. $\angle ABC = 48°$, $\angle ACB = 36°$, find $\angle BAC$

We know that the angles inside of a triangle equal 180°. So,

$\angle ABC+\angle ACB+\angle BAC = 180°$

$48°+36°+\angle BAC = 180°$

$84° + \angle BAC = 180°$

$-84° \quad -84°$

$\angle BAC = 96°$

2. $\angle ACB = 2x$, $\angle BAC = x + 10$, $\angle ABC = 3x + 8$, find x, $\angle ACB$, $\angle BAC$ and $\angle ABC$

We know that the angles inside of a triangle equal 180°. So,

$\angle ABC + \angle ACB + \angle BAC = 180$

$2x + (x + 10) + (3x + 8) = 180$ (Combine like terms)

$6x + 18 = 180$

$-18 \quad -18$

$6x = 162$

$x = 27°$

So, $\angle ACB = 2x = 2(27°) = 54°$

$\angle BAC = x + 10 = 27° + 10° = 37°$

$\angle ABC = 3x + 8 = 3(27°) + 8° = 81° + 8° = 89°$

And, $54° + 37° + 89° = 180°$

3. $\angle ABC = 46°$, $\angle ACB = 42°$, find $\angle BAC$ and $\angle ABD$

We know that the angles inside of a triangle equal 180°. So,

$\angle ABC + \angle ACB + \angle BAC = 180°$

$46° + 42° + \angle BAC = 180°$

$88° + \angle BAC = 180°$

$-88° \quad -88°$

$\angle BAC = 92°$

And we know that $\angle ABD + \angle ABC = 180°$.

$\angle ABD + 46° = 180°$

$-46° \quad -46° \qquad \angle BAC = 134°$

Triangle Inequality Theorem

We have been working with triangles throughout this unit. Now let's explore this question: Can a triangle be formed given any three segment lengths? Your first response is probably “yes”. If you have a piece of uncooked spaghetti at your house, go get two pieces. Break the first piece into three relatively the same length pieces and see if you can form a triangle. Remember to join the pieces end-to-end. It should have worked. Now take the second piece of spaghetti and break two very small pieces off one end of it. (About 1 inch each.) The long piece should still be 4-5 inches long. Now try making a triangle as you just did. Any problems this time? Did your pieces of spaghetti look something like this?

Tape the two sets of spaghetti pieces on a sheet of paper to be turned into your teacher.

Now, you are going to read about the Triangle Inequality Theorem.

Triangle Inequality Theorem: The sum of the measures of any two sides of any triangle is greater than the measure of the third side. In other words, you can pick any two sides measure and when they are added together the sum will be greater than the measure of the third side.

Example: Can the following lengths form a triangle?

1. 8, 6, 2

8 + 6 = 14 > 2 ok

8 + 2 = 10 > 6 ok

6 + 2 = 8 not >8 fails

So, no, can’t form a triangle2. 3, 4, 5

3 + 4 = 7 > 5 ok

3 + 5 = 8 > 5 ok

4 + 5 = 9 > 3 ok

So, yes, can form a triangle

Now practice the Triangle Inequality Theorem.

Can a triangle be formed with the following lengths? Explain each answer.

1. 2", 3", 5"

2. 4 cm, 1 cm, 2 cm

3. 6', 8', 1'

4. 5 m, 5 m, 5 m

5. 2x, 3x, 4x

G. Lesson 6: Altitudes, Medians, and Bisectors

Read the definitions for altitudes, medians, angle bisectors, and perpendicular bisectors of triangles.

Pictures of properties for altitudes, medians, and perpendicular bisectors.

Median

Altitude• In an acute triangle, all of the altitudes are inside the triangle.

• In a right triangle, one of the altitudes is inside the triangle and the other two are the legs of the triangle.• In an obtuse triangle, one of the altitudes is inside the triangle and the other two

are outside the triangle.

Perpendicular bisector

Use the information you learned and your algebra skills to answer the following questions:

1. Segment AD is an altitude.

m∠ ADB = 5x - 10.

m∠ ABD = 2x + 5

Find x, m∠ ABD, and m∠ BAD.2. Segment AD is an angle bisector.

m∠ BAD = 3x - 5

m∠ DAC = x + 29

Find x, m∠ BAD, and

m∠ BAC.

3. Segment AD is median.

BD = 5x + 2

DC = 2x + 8

Find x, BD, and BC.

4. Segment ED is a perpendicular

bisector.

BD = 3x - 3

DC = x + 2

Find x, BD, BC, and m∠ EDC.

5. Write a paragraph for each of the four types of special segments that you have explored in this lesson describing their placement in reference to the triangle as the

triangle changes from acute to right to obtuse.

Chapter-10

Three Dimensional Objects

Pythagorean Theorem

Introduction

The learner will utilize the Pythagorean Theorem, its converse, special right triangles, and the trigonometric functions sine, cosine, and tangent to find missing measures in right triangles, determine the type of triangle according to its angles, and solve problems.

The learner will:

• connect geometric diagrams with algebraic representations.

• use Pythagorean theorem and its converse.

• use right triangle relationships such as trigonometric ratios (45-45-90 and 30-60-90 triangles).

B. Lesson 1: The Pythagorean Theorem

Read the definitions of the Pythagorean Theorem and Pythagorean Triples.

C. Lesson 2: Using the Pythagorean Theorem

Now, finally we are going to work out problems using the Pythagorean Theorem

Converse of the Pythagorean Theorem

This lesson will allow you to see how the Converse of the Pythagorean Theorem can be used as well as more practice using the Triangle Inequality Theorem from the last unit.

Read the definition of the Converse of the Pythagorean Theorem.

The following rules apply to triangles and the Converse of the Pythagorean Theorem.

• In any triangle

$a + b > c$

$b + c > a$

$a + c > b$

• In any triangle, $a < b$, if and only if $\angle A < \angle B$.

• All of a triangles sides are of equal length if and only if all of its angles are equal.

• In a right trianglec

Quadrilaterals

Introduction

The learner will recognize special quadrilaterals and the appropriate properties associated with each quadrilateral and utilize these properties and drawings to apply formulas of area, surface area, and volume for a variety of geometric shapes and solids to calculate these measurements.

The learner will:

• describe, draw, and construct two-dimensional and three-dimensional figures.

• use properties of quadrilaterals such as classification.

• use properties of other polygons.

• use relationships among one-, two-, and three-dimensional measures.

• use perimeter, circumference, and area of planar regions to determine volume and surface area of solids.

• use properties of circles.

B. Lesson 1: Basic Polygons

In this lesson, you will practice naming polygons as well as drawing polygons. Read the definitions for concave, convex, equilateral triangle, heptagon, hexagon, isosceles trapezoid, isosceles triangle, octagon, parallelogram, pentagon, rectangle, right triangle, scalene, and trapezoid, then complete the following assignment. You will need coloring pencils, a ruler, and protractor for the drawing part of this lesson. Assignment:

Draw each of the following polygons: equilateral triangle, heptagon, hexagon, isosceles trapezoid, isosceles triangle (non-equilateral), octagon, parallelogram (not a rectangle or rhombus), pentagon, rectangle, right triangle, scalene triangle, and trapezoid (nonisosceles). Also, write a paragraph for each graph explaining how you created it.

C. Lesson 2: Special Quadrilaterals

Read the definitions for congruent, diagonal, kite, parallelogram, quadrilateral, rectangle, rhombus, square, and trapezoid. You will now explore some specific polygons and special quadrilaterals. (Note: Not every quadrilateral has parallel sides or equal sides!)

The following are Special Quadrilaterals:

Kite (can be a parallelogram because if two pairs of sides are equal, then it is a
rhombus or if the angles are all equal then it is a square)

Parallelogram

Rectangle (also a parallelogram, since it has two pairs of parallel sides)

Rhombus (not always a rectangle because it does not have four right angles and not all four sides of a rectangle have to be equal)

Square (also a rectangle and a parallelogram)

Trapezoid (not a parallelogram because it has only one pair of parallel sides)

The Four Theorems for Parallelograms:

- The diagonal of any parallelogram forms two congruent triangles.
- Both pairs of opposite sides in a parallelogram are congruent.
- Both pairs of opposite angles in a parallelogram are congruent.
- The diagonals of any parallelogram bisect each other.

Ex.

Three Theorems to show a Quadrilateral is a Parallelogram

- If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- If one pair of the opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram.

Example:

Rectangle: Find ∠ ABC, CD, BC, BD, and AC.

Since the image is a rectangle, ∠ ABC = 90

o

, CD = AB = 3”, and BC = AD = 8”. Now,

find BD and AC. We do know that BD = AC by definition of a rectangle, so solve for only one segment and you’ll know both segment lengths.

Use the Pythagorean Theorem to find BD, also labeled as c.

Now let's practice working some problems applying the properties of specialquadrilaterals.

Find each indicated length or angle measure. D. Lesson 3: Computing Area

You to refresh your memory about computing area of two-dimensional shapes to help you with finding the surface area of three-dimensional figures in the next lesson. Most students will have had area and perimeter of two-dimensional figures usually in the seventh and/or the eighth grades. That's been some time ago. So, let's review. Start by reading the definitions for area and perimeter. Then, review the following formulas.

Area and perimeter formulas that you should need:

Circle:

Area = π r

2

Perimeter = 2π r

Ellipse:

Area = π r1r2 Perimeter = 2π

2 2

1 2

(r) ( )

2

+ r

Equilateral Triangle:

Area =

3

4

(a

2

) Perimeter = 3a

Parallelogram:

Area = bh Perimeter = 2a + 2b

Rectangle: Area = ab Perimeter = 2a + 2b

Rhombus:

Area =

1

2

d1d2 Perimeter = a + b + c + d

Square:

Area = s

2

Perimeter = 4s

Trapezoid:

Area =

1

2

h(b1 + b2) Perimeter = a1 + b1 + a2 + b2

Triangle:

Area =

1

2

bh Perimeter = a + b + c

For solid, or 3-d, figures use the following formulas for total area (T) and volume (V).

Cone: T = π rl + π r

2

V =

1

3

π r

2

h

Cylinder:

T = 2π rh + 2π r

2

V = π r

2

h

Pyramid:

T =

1

2

PL V =

1

3

Bh

Where, B is the area of the base and P is the perimeter.For T we are using a regular pyramid which has a base that is a regular polygon

and with lateral faces that are all congruent isosceles triangles.

Right Prism:

T = 2B + Ph V = Bh

where, B = lw and P = 2l + 2w

Sphere:

T = 4π r

2

V =

4

3

π r

3

Example:

Find the volume.

V = lwh

V = (7.5 cm)(3 cm)(2cm) = 45 cm

3Assignment:

Find the area and perimeter from the given information.

1. The side of a square is 2”.

2. The length of a rectangle is 4 cm and the width is 3 cm.

3. The base of a triangle is 6”, its height is 4”, and both legs are 5”.

4. A circle has a diameter of 5 cm.

Three-Dimensional Figures/Polyhedras

Now we are going to switch to polyhedras. Begin by reading the definitions for cube, dodecahedron, icosahedrons, octahedron, polyhedra, tetrahedron, and vertices.Vertices and edges in the polyhedras we will be looking at:

Tetrahedron - has four vertices and six edges.

Octahedron - has six vertices and twelve edges.

Cube – has eight vertices and twelve edges.

Icosahedrons - has twelve vertices and thirty edges.

Dodecahedron - has twenty vertices and thirty edges.

Assignment:

For this assignment you will need scissors, glue or tape, paper, and the given pictures, because you are going to construct the five regular polyhedra that you explored earlier. Print out the following four pictures of the regular polyhedra and make one yourself for a cube, then construct them using your scissors and either tape or glue. A nice way to display your solids is to create a mobile. To create a mobile, you will need some type of frame. A wire clothes hanger works very well. (You will need wire snips to cut it.) You will also need some type of string to hang the solids. Either nylon thread or fishing line works well. Experiment with making your mobile having different levels by using more than one piece of clothes hanger, hanging two solids on one level and three solids on the other level. Work with the solids to see what balances the best.

: Congruent Polygons and Transformations

Unit 6 Introduction

The learner will recognize the conditions needed to prove polygons congruent and/or similar and corresponding parts. Then the learner will use the properties of transformations in connection with congruency and similarity to draw images of figures as reflections, translations, rotations, and dilations.

The learner will:

- connect geometric diagrams with algebraic representations.
- prove triangles and other polygons congruent and similar, and explore corresponding parts relationships.
- use reflections, translations, rotations, and dilations.

B. Lesson 1: Introduction to Congruent Triangles

There are three ways to prove that two triangles are congruent: ASA (Angle-Side-Angle), SAS (Side-Angle-Side), and SSS (Side-Side-Side). Read the definitions for congruent, congruent angles, ASA, SAS, and SSS. For each of them you always get only ONE, unique triangle. Some people will try to prove congruency by SSA (Side-Side-Angle).

This is not possible because with SSA you get two possible triangles with the same measures but theses two triangles are not congruent. Therefore it cannot be used to guarantee congruency for triangles.

Chapter-11

Coordinate Geometry

Similar Triangles

Now we are going to learn about similar triangles. Read the following definitions: ASA part 2, SAS part 2, SSS part 2, and similar polygons. Hopefully, you will quickly see the difference between congruent and similar shapes.

The three Similarity Rules for Triangles are:

- angle-angle similarity
- side-angle-side similarity
- side-side-side similarity

Similarity and Area, Volume, and Scale

This lesson will have you explore how similarity relates to other geometric concepts such as area, volume, and scale drawings. From previous lessons, we know that if two shapes are similar, their corresponding sides have the same ratio and their corresponding angles are equal. But what you may not know is that if two shapes are similar, then their lengths, area, and volumes also have the same ratio. This means that for similar shapes:

- Ratio of lengths = a:b or

a

b

- Ratio of areas = a

2

:b

2

or

2

2

a

b

- Ratio of volumes = a

3

:b

3

or

3

3

a

b

Examples:

1. Given that the two shapes are similar find the missing variable.

The ratios of the areas is 36:81

The ratios of the lengths is 4:x

To solve and compare the ratios we need them to all be to the first powers.

a

2

= 36 and b

2

= 81, to get just a and b take the square root of both sides

a

2

= 36 ⇒

2

a 36 = ⇒ a = 6

b

2

= 81 ⇒

2

b 81 = ⇒ b = 9

Now we have everything to the first power so that we can compare ratio to ratio.

6 4

9 x

= (the single power ratios of area to the single power ratios of length)

To solve use cross products.

6 4

9 x

= ⇒ (6)(x) = (9)(4) ⇒ 6x = 36 ⇒

6x 36

6 6

= ⇒ x = 62. Given that the two shapes are similar what is the ratio of their areas?

The ratios of the volumes is 8:64

To solve and compare the ratios we need them to all be to the first powers.

a

3

= 8 and b

3

= 64, to get just a and b take the cube root of both sides

a

3

= 8 ⇒

3 3 3

a 8 = ⇒ a = 2

b

3

= 64 ⇒

3 3 3

b 64 = ⇒ b = 4

Now we have everything to the first power. The ratio of the lengths is now 2:4. To get the ratios of the areas we just need to square a and b, since all sides are equal this is possible. a

2

= 2

2

= 4 and b

2

= 4

2

= 16

So, the ratio of the areas is 4:16.

Assignment: Solve for the variables.

1.

2.3.

Transformations

This lesson will explore transformations and their connections to congruency and similarity. Read the definitions for reflection, rotation, line of symmetry, transformation, and translation.

Translation: Translate A to the left and down.

Reflection: Reflect B.Symmetry: Show where the given triangle is symmetric by its line of symmetry.

Assignment:

What letters of the alphabet have symmetry? Show their symmetry. (Hint: Some have more than one line of symmetry!)

Answer Key for Assignments in Geometry:

Unit 1 Lesson Answers

Unit 1, Lesson 1

TD = 23, DR = 15

Unit 1, Lesson 2

a. AB = 13, M: (5.5, 1)

b. AB = 10, M: (-7.5, -1.5)

c. AB = 65, M: (1.5, -1)

Unit 1, Lesson 5

Exploration

1. AED and CEB; AEC and DEB

2. They measure the same (they are congruent)

3. AED and DEB; DEB and BEC; BEC and CEA;

CEA and AED

4. The sum is 180 degrees.

Conjectures

1. their measures are equal (the angles are congruent)

2. the sum of their measures is 180 degreesUnit 2 Lesson Answers

Unit 2, Lesson 1

1. straight line, 180

o

2. corresponding angles, varies

3. consecutive interior angles, 180

o

4. corresponding angles, varies

5. straight line, 180

o

6. ∠ EGA and ∠ GHC, ∠ AGH and ∠ CHF, or ∠ BGH and ∠ DHF; varies

7. ∠ AGH and ∠ GHC, 180

o

Unit 2, Lesson 2

Exploration #1

1. alternate interior angles

2. they measure the same (are congruent)

3. alternate interior angles

4. they measure the same (are congruent)

5. same side interior angles

6. 180

o

7. yes

8. same side interior angles

9. 180

o

10. yes

Conjecture

1. alternate interior angles; congruent

2. same side interior angles; supplementary

Exploration #2

1. all pairs are corresponding angles

2. they are congruent (measure the same)

Conjecture

Corresponding angles are congruent (have the same measure).

1. same side interior angles

2. alternate exterior angles

3. alternate interior angles

4. corresponding angles 5. vertical angles

6. linear pairs (supplementary angles)

7. same side interior angles

8. v and w; alternate exterior angles are congruent

9. g and h; corresponding angles are congruent

10. v and w; same side interior angles are supplementary

11. v and w; corresponding angles are congruent

12. g and h; corresponding angles are congruent

13. v and w; alternate interior angles are congruent

The Triangle Inequality Theorem states that the sum of the measures of any two sides of a triangle must always be greater than the measure of the third side.

Unit 3 Lesson 6

1. 20, 45, 45

Solution: $5x - 10 = 90$

o

$+10 \quad +10$

$5x = 100$

$5 \quad\quad 5$

$x = 20$

$\angle ABD = 2(20) + 5 = 45$

$\angle BAD + \angle ADB + \angle ABD = 180$

∠ BAD + 90 + 45 = 180

∠ BAD = 45

2. 17, 46, 92 (Use a similar method to one)

3. 2, 12, 24 (Use a similar method to one)

4. 2.5, 4.5, 9, 90

o

(Use a similar method to one)

5. In a right triangle two of the altitudes are the legs of the triangle, the third altitude and all of the other special segments are in the interior of the triangle. In an obtuse triangle two of the altitudes lie outside the triangle, the third altitude and all of the other special segments are in the interior of the triangle. In an acute triangle all of the special segments lie inside the triangle.

Unit 4 Lesson Answers

Unit 4 Lesson 2

1. 15

2. 13

3. 5

4. 3 10 or 90Unit 4 Lesson 3

1. obtuse, 100 > 25 + 49

2. obtuse, 144 > 64 + 64

3. acute, 16 < 9 + 9

4. right, 100 = 36 + 64

5. acute, x

2

< x

2

+ x

2

Chapter-12

Commercial Mathematics & Probability

Business maths are mathematics used by commercial enterprises to record and manage business operations. Commercial organizations use mathematics in accounting, inventory management, marketing, sales forecasting, and financial analysis. Mathematics typically used in commerce includes elementary arithmetic, elementary algebra, statistics and probability. Business management can be made more effective in some cases by use of more advanced mathematics such as calculus, matrix algebra and linear programming.

University level

In academia, "Business Mathematics" includes mathematics courses taken at an undergraduate level by business students. These courses are slightly less difficult and do not always go into the same depth as other mathematics courses for people majoring in mathematics or science fields. The two most common math courses taken in this form are Business Calculus and Business Statistics. Examples used for problems in these courses are usually real-life problems from the business world.

An example of the differences in coursework from a business mathematics course and a regular mathematics course would be calculus. In a regular calculus course, students would study trigonometric functions. Business calculus would not study trigonometric functions because it would be time-consuming and useless to most business students, except perhaps economics majors. Economics majors who plan to continue economics in graduate school are strongly encouraged to take regular calculus instead of business calculus, as well as linear algebra and other advanced math courses, especially real analysis.

High school

Another meaning of business mathematics, sometimes called commercial math or consumer math, is a group of practical subjects used in commerce and everyday life. In schools, these subjects are often taught to students who are not planning a university education. In the United States, they are typically offered in high schools and in schools that grant associate's degrees.

A U.S. business math course might include a review of elementary arithmetic, including fractions, decimals, and percentages. Elementary algebra is often included as well, in the context of solving practical business problems. The practical applications typically include checking accounts, price discounts, markups and markdowns, payroll calculations, simple and compound interest, consumer and business credit, and mortgages.

Probability

Probability is a measure or estimation of likelihood of occurrence of an event. Probabilities are given a value between 0 (0% chance or *will not happen*) and 1 (100% chance or *will happen*). The higher the degree of probability, the more likely the event is to happen, or, in a longer series of samples, the greater the number of times such event is expected to happen. A simple example is acoin toss that has 0.5 or 50% chance of landing with the "head" side facing up.

These concepts have been given an axiomatic mathematical derivation in probability theory (seeprobability axioms), which is used widely in such areas of study as mathematics, statistics, finance,gambling, science, artificial intelligence/machine learning and philosophy to, for example, draw inferences about the expected frequency of events. Probability theory is also used to describe the underlying mechanics and regularities of complex systems.

When dealing with experiments that are random and well-defined in a purely theoretical setting (like tossing a fair coin), probabilities describe the statistical number of outcomes considered divided by the number of all outcomes (tossing a fair coin twice will yield head-head with probability 1/4, because the four outcomes head-head, head-tails, tails-head and tails-tails are equally likely to occur). When it comes to practical application however there are two major conflicting categories of **probability interpretations**, whose adherents possess different views about the fundamental nature of probability:

1. Objectivists assign numbers to describe some objective or physical state of affairs. The most popular version of objective probability is frequentist probability, which claims that the probability of a random event denotes the *relative frequency of occurrence* of an experiment's outcome, when repeating the experiment. This interpretation considers probability to be the relative frequency "in the long run" of outcomes. A modification of this is propensity probability, which interprets probability as the tendency of some experiment to yield a certain outcome, even if it is performed only once.

2. Subjectivists assign numbers per subjective probability, i.e., as a degree of belief. The degree of belief has been interpreted as, "the price at which you would buy or sell a bet that pays 1 unit of utility if E, 0 if not E." The most popular version of subjective probability is Bayesian probability, which includes expert knowledge as well as experimental data to produce probabilities. The expert knowledge is represented by some (subjective) prior probability distribution. The data is incorporated in a likelihood function. The product of the prior and the likelihood, normalized, results in a posterior probability distribution that incorporates all the information known to date. Starting from arbitrary, subjective probabilities for a group of agents, some Bayesians [s]claim that all agents will eventually have sufficiently similar assessments of probabilities, given enough evidence (see Cromwell's rule).

Etymology

The word Probability derives from the Latin probabilitas, which can also mean probity, a measure of the authority of a witness in alegal case in Europe, and often correlated with the witness's nobility. In a sense, this differs much from the modern meaning of *probability*, which, in contrast, is a measure of the weight of empirical evidence, and is arrived at from inductive reasoning and statistical inference.

History

The scientific study of probability is a modern development. Gambling shows that there has been an interest in quantifying the ideas of probability for millennia, but exact mathematical descriptions arose much later. There are reasons of course, for the slow development of the mathematics of probability. Whereas games of chance provided the impetus for the mathematical study of probability, fundamental issues are still obscured by the superstitions of gamblers.

According to Richard Jeffrey, "Before the middle of the seventeenth century, the term 'probable' (Latin*probabilis*) meant *approvable*, and was applied in that sense, univocally, to opinion and to action. A probable action or opinion was one such as sensible people would undertake or hold, in the circumstances." However, in legal contexts especially, 'probable' could also apply to propositions for which there was good evidence.

Aside from elementary work by Gerolamo Cardano in the 16th century, the doctrine of probabilities dates to the correspondence of Pierre de Fermat and Blaise

Pascal (1654). Christiaan Huygens (1657) gave the earliest known scientific treatment of the subject. Jakob Bernoulli's Ars Conjectandi(posthumous, 1713) and Abraham de Moivre's Doctrine of Chances (1718) treated the subject as a branch of mathematics. See Ian Hacking's The Emergence of Probability and James Franklin'sThe Science of Conjecture for histories of the early development of the very concept of mathematical probability.

The theory of errors may be traced back to Roger Cotes's Opera Miscellanea (posthumous, 1722), but a memoir prepared by Thomas Simpson in 1755 (printed 1756) first applied the theory to the discussion of errors of observation. The reprint (1757) of this memoir lays down the axioms that positive and negative errors are equally probable, and that certain assignable limits define the range of all errors. Simpson also discusses continuous errors and describes a probability curve.

The first two laws of error that were proposed both originated with Pierre-Simon Laplace. The first law was published in 1774 and stated that the frequency of an error could be expressed as an exponential function of the numerical magnitude of the error, disregarding sign. The second law of error was proposed in 1778 by Laplace and stated that the frequency of the error is an exponential function of the square of the error. The second law of error is called the normal distribution or the Gauss law. "It is difficult historically to attribute that law to Gauss, who in spite of his well-known precocity had probably not made this discovery before he was two years old."

Daniel Bernoulli (1778) introduced the principle of the maximum product of the probabilities of a system of concurrent errors.

Carl Friedrich Gauss

Adrien-Marie Legendre (1805) developed the method of least squares, and introduced it in his Nouvelles méthodes pour la détermination des orbites des comètes (New Methods for Determining the Orbits of Comets). In ignorance of Legendre's contribution, an Irish-American writer, Robert Adrain, editor of "The Analyst" (1808), first deduced the law of facility of error,

$$\phi(x) = ce^{-h^2 x^2},$$

where h is a constant depending on precision of observation, and c is a scale factor ensuring that the area under the curve equals 1. He gave two proofs, the second being essentially the same as John Herschel's (1850). Gauss gave the first proof that seems to have been known in Europe (the third after Adrain's) in 1809. Further proofs were given by Laplace (1810, 1812), Gauss (1823),James Ivory (1825, 1826), Hagen (1837), Friedrich Bessel (1838), W. F. Donkin (1844, 1856), and Morgan Crofton (1870). Other contributors were Ellis (1844), De Morgan (1864), Glaisher (1872), and Giovanni Schiaparelli (1875). Peters's (1856) formulafor *r*, the probable error of a single observation, is well known.

In the nineteenth century authors on the general theory included Laplace, Sylvestre Lacroix (1816), Littrow (1833), Adolphe Quetelet(1853), Richard Dedekind (1860), Helmert (1872), Hermann Laurent (1873), Liagre, Didion, and Karl Pearson. Augustus De Morganand George Boole improved the exposition of the theory.

Andrey Markov introducedthe notion of Markov chains (1906), which played an important role in stochastic processestheory and its applications. The modern theory of probability based on the measure theory was developed by Andrey Kolmogorov(1931).

On the geometric side (see integral geometry) contributors to *The Educational Times* were influential (Miller, Crofton, McColl, Wolstenholme, Watson, and Artemas Martin).

Theory

Like other theories, the theory of probability is a representation of probabilistic concepts in formal terms—that is, in terms that can be considered separately from their meaning. These formal terms are manipulated by the rules of mathematics and logic, and any results are interpreted or translated back into the problem domain.

There have been at least two successful attempts to formalize probability, namely the Kolmogorov formulation and the Coxformulation. In Kolmogorov's formulation (see probability space), sets are interpreted as events and probability itself as a measure on a class of sets. In Cox's theorem, probability is taken as a primitive (that is, not further analyzed) and the emphasis is on constructing a consistent assignment of probability values to propositions. In both cases, the laws of probability are the same, except for technical details.

Governments apply probabilistic methods in environmental regulation, where it is called pathway analysis

A good example is the effect of the perceived probability of any widespread Middle East conflict on oil
prices—which have ripple effects in the economy as a whole. An assessment by a commodity trader that a war is more likely vs. less likely sends prices up or down, and signals other traders of that opinion. Accordingly, the probabilities are neither assessed independently nor necessarily very rationally. The theory of behavioral financeemerged to describe the effect of such groupthink on pricing, on policy, and on peace and conflict.

The discovery of rigorous methods to assess and combine probability assessments has changed society. It is important for most citizens to understand how probability assessments are made, and how they contribute to decisions.

Another significant application of probability theory in everyday life is reliability. Many consumer products, such as automobiles and consumer electronics, use reliability theory in product design to reduce the probability of failure. Failure probability may influence a manufacturer's decisions on a product's warranty.

The cache language model and other statistical language models that are used in natural language processing are also examples of applications of probability theory.

Mathematical treatment

Consider an experiment that can produce a number of results. The collection of all results is called the sample space of the experiment. The power set of the sample space is formed by considering all different collections of possible results. For example, rolling a die can produce six possible results. One collection of possible results gives an odd number on the die. Thus, the subset {1,3,5} is an element of the power set of the sample space of die rolls. These collections are called "events." In this case, {1,3,5} is the event that the die falls on some odd number. If the results that actually occur fall in a given event, the event is said to have occurred.

A probability is a way of assigning every event a value between zero and one, with the requirement that the event made up of all possible results (in our example, the event {1,2,3,4,5,6}) is assigned a value of one. To qualify as a probability, the assignment of values must satisfy the requirement that if you look at a collection of mutually exclusive events (events with no common results, e.g., the events {1,6}, {3}, and {2,4} are all mutually exclusive), the probability that at least one of the events will occur is given by the sum of the probabilities of all the individual events.

The probability of an event *A* is written as P(*A*), p(*A*) or Pr(*A*). This mathematical definition of probability can extend to infinite sample spaces, and even uncountable sample spaces, using the concept of a measure.

The *opposite* or *complement* of an event *A* is the event [not *A*] (that is, the event of *A* not occurring); its probability is given byP(not *A*) = 1 - P(*A*). As an example, the chance of not rolling a six on a six-sided die is 1 – (chance of rolling a six) $= 1 - \tfrac{1}{6} = \tfrac{5}{6}.$ See Complementary event for a more complete treatment.

If two events *A* and *B* occur on a single performance of an experiment, this is called the intersection or joint probability of *A* and *B*, denoted as $P(A \cap B).$

Independent probability

If two events, *A* and *B* are independent then the joint probability is

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B),$$

for example, if two coins are flipped the chance of both being heads is $\tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{4}.$

Mutually exclusive

If either event *A* or event *B* or both events occur on a single performance of an experiment this is called the union of the events *A* and*B* denoted as $P(A \cup B).$ If two events are mutually exclusive then the probability of either occurring is

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B).$$

For example, the chance of rolling a 1 or 2 on a six-sided die is $P(1 \text{ or } 2) = P(1) + P(2) = \tfrac{1}{6} + \tfrac{1}{6} = \tfrac{1}{3}.$

Not mutually exclusive

If the events are not mutually exclusive then

$$\mathrm{P}\left(A \text{ or } B\right) = \mathrm{P}\left(A\right) + \mathrm{P}\left(B\right) - \mathrm{P}\left(A \text{ and } B\right).$$

For example, when drawing a single card at random from a regular deck of cards, the chance of getting a heart or a face card (J,Q,K) (or one that is both) is $\frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{11}{26}$, because of the 52 cards of a deck 13 are hearts, 12 are face cards, and 3 are both: here the possibilities included in the "3 that are both" are included in each of the "13 hearts" and the "12 face cards" but should only be counted once.

Conditional probability

Conditional probability is the probability of some event *A*, given the occurrence of some other event *B*. Conditional probability is written $\mathrm{P}(A \mid B)$, and is read "the probability of *A*, given *B*". It is defined by

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}.$$

If $\mathrm{P}(B) = 0$ then $\mathrm{P}(A \mid B)$ is formally undefined by this expression. However, it is possible to define a conditional probability for some zero-probability events using a σ-algebra of such events (such as those arising from a continuous random variable).

For example, in a bag of 2 red balls and 2 blue balls (4 balls in total), the probability of taking a red ball is however, when taking a second ball, the probability of it being either a red ball or a blue ball depends on the ball previously taken, such as, if a red ball was taken, the probability of picking a red ball again would be since only 1 red and 2 blue balls would have been remaining.

Inverse probability

In probability theory and applications, Bayes' rule relates the odds of event A_1 to event A_2, before (prior to) and after (posterior to)conditioning on another event B. The odds to event is simply the ratio of the probabilities of the two events. When arbitrarily many events are of interest, not just two, the rule can be rephrased as posterior is proportional to prior times likelihood, where the proportionality symbol means that the left hand side is proportional to (i.e., equals a constant times) the right hand side as A varies, for fixed or given B (Lee, 2012; Bertsch McGrayne, 2012). In this form it goes back to Laplace (1774) and to Cournot (1843); see Fienberg (2005). See Inverse probability and Bayes' rule.

Relation to randomness

In a deterministic universe, based on Newtonian concepts, there would be no probability if all conditions were known (Laplace's demon), (but there are situations in which sensitivity to initial conditions exceeds our ability to measure them, i.e. know them). In the case of a roulette wheel, if the force of the hand and the period of that force are known, the number on which the ball will stop would be a certainty (though as a practical matter, this would likely be true only of a roulette wheel that had not been exactly levelled — as Thomas A. Bass' Newtonian Casino revealed). Of course, this also assumes knowledge of inertia and friction of the wheel, weight, smoothness and roundness of the ball, variations in hand speed during the turning and so forth. A probabilistic description can thus be more useful than Newtonian mechanics for analyzing the pattern of outcomes of repeated rolls of a roulette wheel. Physicists face the same situation in kinetic theory of gases, where the system, while deterministic *in principle*, is so complex (with the number of molecules typically the order of magnitude of Avogadro constant $6.02 \cdot 10^{23}$) that only a statistical description of its properties is feasible.

Probability theory is required to describe quantum phenomena. A revolutionary discovery of early 20th century physics was the random character of all physical processes that occur at sub-atomic scales and are governed by the laws of quantum mechanics. The objective wave function evolves deterministically but, according to the Copenhagen interpretation, it deals with probabilities of observing, the outcome being explained by a wave function collapse when an observation is made. However, the loss of determinismfor the sake of instrumentalism did not meet with universal approval. Albert Einstein famously remarked in a letter to Max Born: "I am convinced that God does not play dice". Like Einstein, Erwin Schrödinger, who discovered the wave function, believed quantum mechanics is a statistical approximation of an underlying deterministic reality. In modern interpretations, quantum decoherence accounts for subjectively probabilistic behavior.

Probability axioms

A random experiment results in some outcome. The set of all possible outcomes is called the sample space. A subset of the sample space is called an event. The probability of an event satisfies the following axioms.

Basic Axioms of Probability

Lets denote the sample space of a random experiment with S and A be any event which is subset of S

For any event A, $P(A) \geq 0$

$P(S)=1$

For a finite or infinite sequence of disjoint events A_1, A_2, A_3 ....

$$\mathbf{P(\cup_i A_i) = \textstyle\sum_i P(A_i)}$$

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